Logic and its Applications to the Sciences

Joshua Sack

Institute for Logic, Language, and Computation
University of Amsterdam

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Overview

Logical languages and semantics can be used to help us reason about problems in science. We will focus on three examples of such problems:

- Quantum theory
- Computation and time
- Social behavior

Quantum Theory

Locality

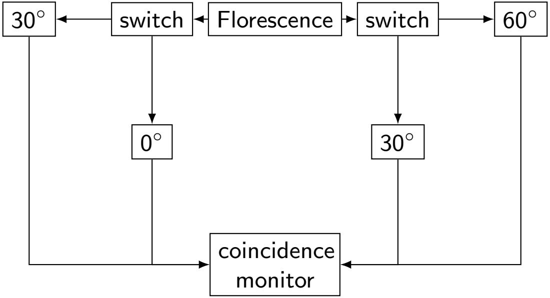
- Locality: A property, assumed by classical (as opposed to quantum) physics "that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past."

  J. Bell. Physics 1, 195 (1964).

- Bell inequalities: Certain inequalities (due to Bell's theorem) that are derivable from the assumption of classical physics. If Bell inequalities are violated, then so are the assumptions of classical physics.
- Logical Bell inequalities: A logical formulation of Bell inequalities that covers a wide range of non-local behaviors.

  S. Abramsky, L. Hardy. Phys. Review A 85, 062114(2012).

Alain Aspect style experiment



Experiment results

	(B, B)	(P, B)	(B, P)	(P, P)
$(30^\circ, 30^\circ)$	$1/2$	0	0	$1/2$
$(30^\circ, 60^\circ)$	$3/8$	$1/8$	$1/8$	$3/8$
$(0^\circ, 30^\circ)$	$3/8$	$1/8$	$1/8$	$3/8$
$(0^\circ, 60^\circ)$	$1/8$	$3/8$	$3/8$	$1/8$

- P - passes through
- B - blocked
- The numbers represent the fraction of pairs of photons with the behavior given by the column.

Game

T. Maudlin. Quantum Non-Locality and Relativity. Blackwell Publishing, 1994.

- You and your partner start in a room (the Calcium atom exhibiting florescence) and make a plan
- You go to separate rooms (and can no longer interact with each other)
- Each of you is told an angle and then asked whether you pass or will be blocked:
- These previous three steps are repeated an arbitrary number of times

Goal is to have guesses match the experimental data with the repeated plays.

Classical (non-quantum) physics assumes particles must interact only by local means, thus conforming to the goal of this game.

Propositional Language

Consider the set of "atomic propositions":

$$\text{AtProp} = \{L30, L0, R30, R60\}$$

Each round of the game, you and your partner must say for some $p \in \text{AtProp}$ whether p is "true" (blocking) or "false" (passing).

The set of propositional formulas is the smallest set satisfying

- any $p \in \text{AtProp}$ is a formula
- if φ is a formula, then $\neg\varphi$ ("not φ") is a formula
- if φ, ψ are formulas, then $\varphi \wedge \psi$ ("and"), $\varphi \vee \psi$ ("or"), $\varphi \to \psi$ ("only if") are formulas.

After determining which atomic propositions are true, we can determine for each formula whether it is true.

Example

Suppose $V : \text{AtProp} \to \{t, f\}$, such that

- $V(L0) = f$
- $V(L30) = f$
- $V(R30) = t$
- $V(R60) = t$

From this, we can extend v to truth values for other formulas, such as

- $V(\neg R30) = f$
- $V(L0 \wedge R30) = f$
- $V(L0 \vee R30) = t$

Logical Bell Inequalities

Let

- $\varphi_1, \ldots, \varphi_n$ be formulas of propositional logic.
- p_i for $1 \leq i \leq n$ each be the probability of φ_i being true.

If the φ_i are jointly contradictory ($V(\bigvee_i \neg\varphi_i) = t$ for every V), then

$$1 = \Pr(\bigvee_i \neg\varphi_i) \leq \sum_i \Pr(\neg\varphi_i) = \sum_i (1 - p_i) = n - \sum_i p_i.$$

Thus

$$\sum_i p_i \leq n - 1$$

Violation of the Bell inequalities

	(B,B)	(P,B)	(B,P)	(P,P)
$(30^\circ,30^\circ)$	1/2	0	0	1/2
$(30^\circ,60^\circ)$	3/8	1/8	1/8	3/8
$(0^\circ,30^\circ)$	3/8	1/8	1/8	3/8
$(0^\circ,60^\circ)$	1/8	3/8	3/8	1/8

Consider

$$\varphi_1 = (L30 \wedge R30) \vee (\neg L30 \wedge \neg R30) \qquad p_1 = 1$$
$$\varphi_2 = (L30 \wedge R60) \vee (\neg L30 \wedge \neg R60) \qquad p_2 = 3/4$$
$$\varphi_3 = (L0 \wedge R30) \vee (\neg L0 \wedge \neg R30) \qquad p_3 = 3/4$$
$$\varphi_4 = (L0 \wedge \neg R60) \vee (\neg L0 \wedge R60) \qquad p_4 = 3/4$$

For all assignment of truth values to AtProp, $\bigwedge_i \varphi_i$ is false (jointly contradictory). But $\sum_i p_i = 3 + 1/4 > 3 = n - 1$

Hardy's model

Quantum theory supports the following (each row still adds to one, but the constraints are looser):

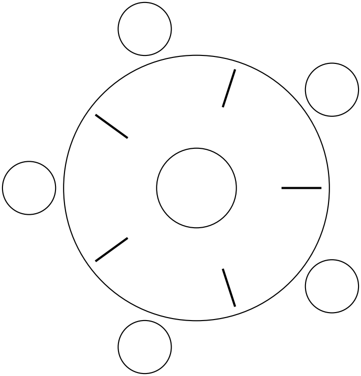
	(t,t)	(f,t)	(t,f)	(f,f)
(a,b)	>0	>0	>0	>0
(a',b)	$=0$	>0	>0	>0
(a,b')	$=0$	>0	>0	>0
(a',b')	>0	>0	>0	$=0$

$$\varphi_1 = a \wedge b \qquad p_1 > 0$$
$$\varphi_2 = \neg(a' \wedge b) \qquad p_2 = 1$$
$$\varphi_3 = \neg(a \wedge b') \qquad p_3 = 1$$
$$\varphi_4 = a' \vee b' \qquad p_4 = 1$$

The φ_i are jointly contradictory. But $\sum_i p_i > 3 = n - 1$.

Computation and Time

Five philosophers sit around a round dining table, with a big bowl of rice in the middle. There is one chopstick to the right of each philosopher.



Philosophers states

Possible states of a philosopher:

- e eating rice (but must have two chopsticks in hand)
- t thinking (but must have no chopsticks in hand)
- w_r waiting for right chopstick
  (must hold left chopstick, but not right)
- w_l waiting for left chopstick
  (must hold right chopstick, but not left)
- r_r returning right chopstick
  (must hold right chopstick, but not left)
- r_l returning left chopstick
  (must hold left chopstick, but not right)

A global state is a tuple $(s_1, s_2, s_3, s_4, s_5)$ of local states for each philosopher, that is compatible with the constraints above.

Example

(t, e, w_l, t, t) could be a global state, but (e, e, t, t, t) does not satisfy the constraints.

Actions

The following actions can be performed of each chopstick

- p_l being picked up by philosopher to the left
- p_r being picked up by philosopher to the right
- t_l being returned by philosopher to the left
- t_l being returned by philosopher to the left
- ϵ no action

A concurrent action is a tuple of local actions:

Example

$(p_r, p_r, p_r, p_r, p_r)$ is the concurrent action where every philosopher picks up the chopstick to her left.

Computation, Deadlock, and Fairness

Philosophers as computational processes

Suppose each philosopher follows a strict protocol. For example, if a philosopher picks up one chopstick, she will wait until the other is available before doing anything else.

- Deadlock occurs if no further concurrent actions are possible
  Example: after everyone picks up the chopstick on their right and waits indefinitely for the one on the left (no further concurrent actions are possible).
- Fairness every philosopher may have a chance to eat infinitely often (fair allocation of computational resources to the processes)

A Labelled transition system (with properties) is a tuple

$$M = (S, \text{Act}, \text{AtProp}, \{R_a\}_{a \in \text{Act}}, V)$$

- S is a set of (global) states
- AtProp set of atomic properties:
  Example: e_3 (third philosopher is eating).
- Act set of (concurrent) actions:
- R_a is a relation on S.
  ($sR_a s'$ iff a can be performed at s and result in s')
- $V : \text{AtProp} \to \mathcal{P}(S)$ is a function mapping each $p \in \text{AtProp}$ to a set of states in S (representing where p is true).

Paths

A path is a sequence of states $\mathbf{s} = s_0, s_1, s_2, s_3, \ldots,$ where for each i, there is a concurrent action a such that

$$s_i R_a s_{i+1}.$$

The path can be finite or infinite.

- let $\ell(\mathbf{s})$ be the length of $\mathbf{s}$ (it would be ∞ if infinite).
- let $\mathbf{s}_{\geq n} = s_n, s_{n+1}, \ldots$ be the suffix of $\mathbf{s}$.

Temporal logic language

The set of temporal logic (TL) formulas is the smallest set such that

- Any atomic property $p \in \text{AtProp}$ is a formula and $\bot$ ("false") is a formula
- if φ is a formula, so is $\neg\varphi$, $X\varphi$ ("next time φ"), $G\varphi$ ("always (globally) will be φ"), and $F\varphi$ ("eventually φ") are formulas
- if φ, ψ are formulas, then $\varphi \wedge \psi$ are formulas.

Those components in red are those not already in propositional logic.

Temporal Logic Semantics

Define truth of a formula on a path $\mathbf{s} = s_0, \ldots$ (finite or infinite) of a labelled transition system by a relation $\models$:

$\mathbf{s} \models p$	$\Leftrightarrow$	$s_0 \in V(p)$
		(The first state along the path has p)
$\mathbf{s} \models \bot$		never
$\mathbf{s} \models \neg\varphi$	$\Leftrightarrow$	$\mathbf{s} \not\models \varphi$
$\mathbf{s} \models \varphi \wedge \psi$	$\Leftrightarrow$	$\mathbf{s} \models \varphi$ and $\mathbf{s} \models \psi$
$\mathbf{s} \models X\varphi$	$\Leftrightarrow$	if $\ell(\mathbf{s}) > 1$, then $\mathbf{s}_{\geq 1} \models \varphi$
		(The next state (if exists) satisfies φ)
$\mathbf{s} \models G\varphi$	$\Leftrightarrow$	for all $0 < n < \ell(\mathbf{s})$, $\mathbf{s}_{\geq n} \models \varphi$
		(φ in all future states)
$\mathbf{s} \models F\varphi$	$\Leftrightarrow$	for some $0 < n < \ell(\mathbf{s})$, $\mathbf{s}_{\geq n} \models \varphi$
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- Note that $(s_0) \models X\bot$ holds (vacuously) true
- Note that $(s_0, s_1, s_2, s_3) \models XX\varphi$ iff $(s_2, s_3) \models \varphi$

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- Note that $(s_0, s_1, s_2, s_3) \models XX\varphi$ iff $(s_2, s_3) \models \varphi$

Expressing Deadlock and Fairness

let AtProp $= \{e_0, e_1, e_2, e_3, e_4\}$, where e_k is the property that k is eating in the current state.

- existence of deadlock is expressed using

$$\text{deadlock} \equiv X\bot \vee FX\bot$$

either the current state is a dead-end, or there will be a dead-end sometime in the future

- fairness is expressed using

$$\text{fairness} \equiv \neg\text{deadlock} \wedge \bigwedge_{k=0}^{4} GFe_k$$

There is no deadlock, and for every agent, at every point in the future, that agent will be able to eat at some later point.

Some classical problems involving temporal logic

- Model Checking: Given a logic, formula, and "model" (labelled transition system), determine whether the formula is true in the model.
- Compositional Reasoning: Find processes for components (the individual philosophers) and compose them into a labelled transition system. Express using a logic both properties of the components and properties of the composed system.

Social Phenomena

Urn example and Informational cascades

Example

- A room has either one of two urns:
  - Majority White: $\{W, W, B\}$
  - Majority Black: $\{B, B, W\}$.
- People line up to enter the room one-by-one to:
  - Draw a ball (observe and replace)
  - Write down a guess (for all to see) as to which urn it is

- In forming a guess, agents take into account:
  - The outcome (ball) of their draw
  - The guesses (which urn) that came before
- A Cascade develops if agents' conclusions are dominated by guesses that came before
  - False cascade: a cascade where agents' conclusions do not match the real situation
  - Correct cascade: a cascade where agents' conclusions do match the real situation

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Goal

We want a logic (to reason about cascades and related examples) with

- Probabilistic components (which urn is more likely)
- Epistemic components (beliefs agents have about each other)
- Common knowledge (of the rules of the example)
- Dynamic updates (to model how agents' views change)

The Probabilistic Logic of Communication and Change is the synthesis of variants of the following logics:

- Logic of Communication and Change for Dynamics of Common Knowledge

van Benthem, van Eijck, and Kooi. Logic of Communication and Change. *Information and Computation* 204(11):1620–1662, 2006.

- Dynamic Epistemic Probabilistic Logic for Dynamics of Probabilities

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Syntax

Examples

- $.6 \cdot P_a(p) + .2 \cdot P_a([b]p) \geq .5$
- $[(a \cup b)^*]p$ It is common knowledge among a and b that p.
- $[e][a]p$ After informational event e, a would believe p.

PLCC

Language of the Probabilistic Logic of Communication and Change:

$$\phi ::= true \mid p \mid \neg\phi \mid \phi \wedge \phi' \mid [\pi]\phi \mid [e]\phi \mid t_a \geq \beta \quad \text{(formulas)}$$

$$t_a ::= \alpha \cdot P_a(\phi) \mid t_a + t'_a \quad \text{(probability terms)}$$

$$\pi ::= a \mid \pi;\pi' \mid \pi \cup \pi' \mid \pi^* \mid ?\phi \quad \text{(epistemic programs)}$$

$p \in At$ (atomic proposition) such as the urn has majority white
$a \in Ag$ (agent)
$\alpha, \beta \in \mathbb{Q}$ (rational numbers)
$e \in E$ (epistemic update event) such as drawing from urn

Static sublanguage

PE-PDL

Language of Probabilistic Epistemic Propositional Dynamic Logic:
Same as PLCC, but with dynamic formulas $[e]\varphi$ removed.

Definition (Bayesian Kripke models)

$M = (S, \sim, \mu, V)$ where:

- S - (non-empty) set of states
- $\sim$ - equivalence relations $\sim_a$ on S, for each agent a
- μ - indexed probability functions $\mu_a : S \to (S \to [0,1])$, with values denoted $\mu_a^s(s')$, and satisfying:
  - State-determined probability (SDP)
    each agent knows her probability
    if $s \sim_a t$, then $\mu_a^s(s') = \mu_a^t(s')$ for all $s' \in S$
  - Consistency (CONS)
    consistency of probabilities with knowledge
    $\mu_a^s(t) = 0$ if $s \not\sim_a t$
  - Caution (CAUT) agents are cautious
    $s \not\sim_a t$ if $\mu_a^s(t) = 0$
  - Probability (PROB) μ_a^s is a probability mass function.
    $\sum_{t \in S} \mu_a^s(t) = 1$
- $V : At \to \mathcal{P}(S)$ a valuation function.

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Select semantics of PE-PDL

$$
\begin{array}{lll}
M, s \models p & \text{iff} & s \in V(p) \\
M, s \models [\pi]\phi & \text{iff} & M, t \models \phi \text{ whenever } s R_\pi t \\
M, s \models \sum_{j=1}^{n} \alpha_j P_a(\phi_j) \geq \beta & \text{iff} & \sum_{j=1}^{n} \alpha_j \cdot \mu_a^s(\phi_j) \geq \beta
\end{array}
$$

where $\mu_a^s(\phi_j)$ abbreviates $\sum_{s' \in S, s' \models \phi_j} \mu_a^s(s')$,

and R_π is a binary relation given by

$$
\begin{array}{lll}
s R_a t & \text{iff} & s \sim_a t \\
s R_{\pi_1 \cup \pi_2} t & \text{iff} & s(R_{\pi_1} \cup R_{\pi_2}) t \\
s R_{\pi_1 ; \pi_2} t & \text{iff} & s R_{\pi_1} ; R_{\pi_2} t \;\; (\text{there is } w \text{, such that } s R_{\pi_1} w \text{ and } w R_{\pi_2} t) \\
s R_{\pi^*} t & \text{iff} & s (R_\pi)^* t \;\; ((R_\pi)^* \text{ the reflexive transitive closure of } R_\pi) \\
s R_{?\phi} t & \text{iff} & s = t \text{ and } M, s \models \phi
\end{array}
$$

Event model

Definition (Event Models)

$A = (E, \sim, \Phi, \text{pre}, \text{sub})$ where:

- E - the (finite non-empty) set of epistemic update events.
- $\sim$ - equivalence relations $\sim_a$ on E for each agent a.
- Φ - finite pairwise inconsistent set of precondition formulas
- pre - functions $\text{pre}_a : \Phi \to (E \to [0,1])$ for each $a \in Ag$, $\text{pre}_a(\phi)$ is a subjective occurrence probability function ($\sum_{e \in E} \text{pre}_a(\phi)(e) = 1$)

$$\text{PRE} : E \to \mathcal{P}(\Phi) \qquad \text{PRE} : e \mapsto \{\phi \mid \text{pre}(\phi)(e) > 0\}$$

$$\text{pre}_a(e \mid s) = \begin{cases} \text{pre}_a(\phi)(e) & \phi \in \Phi, M, s \models \phi \\ 0 & \text{there is no such } \phi \end{cases}$$

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Update product

Definition (Product Update)

$M = (S, \sim, \mu, V)$ Bayesian Kripke model
$A = (E, \sim, \Phi, \text{pre}, \text{sub})$ event model
$M \otimes A = (S \otimes E, \sim, \mu, V)$ update product

- $S \otimes E \stackrel{\text{def}}{=} \{(s, e) \mid s \in S, e \in E, (M, s) \models \bigvee \text{PRE}(e)\}$.
- $(s, e) \sim_a (s', e')$ iff $s \sim_a s'$ and $e \sim_a e'$.
- Let $D \stackrel{\text{def}}{=} \sum_{(s',e') \sim_a (w,g)} (\mu_a^w(s') \cdot \text{pre}_a(e' \mid s'))$, and put:

$$\mu_a^{(w,g)}(s, e) \stackrel{\text{def}}{=} \begin{cases} \frac{\mu_a^w(s) \cdot \text{pre}_a(e|s)}{D} & \text{if } (s, e) \sim_a (w, g) \\ 0 & \text{otherwise} \end{cases}$$

  (Note that $D \neq 0$ for $(w, g) \in S \otimes E$.)
- $V(p) = \{(s, e) \mid M, s \models \text{sub}(e)(p)\}$

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- $V(p) = \{(s, e) \mid M, s \models \text{sub}(e)(p)\}$

Semantics of PLCC

The semantics of PLCC is the same as for PE-PDL, but with the following extra clause:

$$M, s \models [e]\phi \quad \text{iff} \quad M, s \models \bigvee \mathrm{PRE}(e) \text{ then } M \times A, (s, e) \models \phi,$$

where e is an event in action model A

Urn example with formulas

Basic elements of the language

- $Ag \stackrel{\text{def}}{=} \{1, \ldots, n\}$ (agents)
- $At \stackrel{\text{def}}{=} \{\mathrm{MW}, \mathrm{MB}\} \cup \{\mathrm{DW}_i, \mathrm{DB}_i, \mathrm{W}_i, \mathrm{B}_i\}_{i \in Ag}$ (atoms)
- $E \stackrel{\text{def}}{=} \{\mathrm{dw}_i, \mathrm{db}_i, \mathrm{w}_i, \mathrm{b}_i\}_{i \in Ag}$ (events)

Some useful formulas and sets:

$$\chi_i \stackrel{\text{def}}{=} (\mathrm{MW} \vee \mathrm{MB}) \wedge \bigwedge_{j<i}(\mathrm{DW}_j \vee \mathrm{DB}_j) \wedge \bigwedge_{j<i}(\mathrm{W}_j \vee \mathrm{B}_j)) \wedge \bigwedge_{p \in At_{\geq i}} \neg p$$

The situation right before i draws

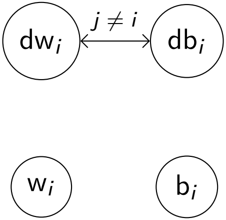
$$\chi_i^D \stackrel{\text{def}}{=} (\mathrm{MW} \vee \mathrm{MB}) \wedge \bigwedge_{j \leq i}(\mathrm{DW}_j \vee \mathrm{DB}_j) \wedge \bigwedge_{j<i}(\mathrm{W}_j \vee \mathrm{B}_j) \wedge \neg(\mathrm{W}_i \vee \mathrm{B}_i) \wedge \bigwedge_{p \in At_{>i}} \neg p$$

The situation right before i writes

$$At_{\geq i} \stackrel{\text{def}}{=} \{\mathrm{DW}_j, \mathrm{DB}_j, \mathrm{W}_j, \mathrm{B}_j\}_{i \leq j \leq n}$$

$$At_{>i} \stackrel{\text{def}}{=} \{\mathrm{DW}_j, \mathrm{DB}_j, \mathrm{W}_j, \mathrm{B}_j\}_{i < j \leq n}$$

$\text{pre}(\psi_i^W)(\text{dw}_i) = \frac{2}{3}$
$\text{pre}(\psi_i^B)(\text{dw}_i) = \frac{1}{3}$



$\text{pre}(\psi_i^W)(\text{db}_i) = \frac{1}{3}$
$\text{pre}(\psi_i^B)(\text{db}_i) = \frac{2}{3}$

$\text{pre}(\phi_i^W)(\text{w}_i) = 1$
$\text{pre}(\phi_i^B)(\text{w}_i) = 0$

$\text{pre}(\phi_i^W)(\text{b}_i) = 0$
$\text{pre}(\phi_i^B)(\text{b}_i) = 1$

$\psi_i^W \stackrel{\text{def}}{=} \text{MW} \wedge \chi_i$
The urn has majority white and
it is right before agent i draws a ball.

$\psi_i^B \stackrel{\text{def}}{=} \text{MB} \wedge \chi_i$

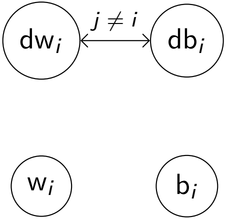
$\phi_i^W \stackrel{\text{def}}{=} P_i(\text{MW}) > P_i(\text{MB}) \vee (\text{DW}_i \wedge P_i(\text{MW}) = P_i(\text{MB})) \wedge \chi_i^D$
i either considers the majority white urn more likely or
i considers them equally likely but had just drawn white.

$\phi_i^B \stackrel{\text{def}}{=} P_i(\text{MB}) > P_i(\text{MW}) \vee (\text{DB}_i \wedge P_i(\text{MW}) = P_i(\text{MB})) \wedge \chi_i^D$

Depiction of Event Model: updates of atoms

$\mathrm{sub}(\mathrm{dw}_i, p) = \begin{cases} \chi_i & p = \mathrm{DW}_i \\ p & p \neq \mathrm{DW}_i \end{cases}$

$\mathrm{sub}(\mathrm{db}_i, p) = \begin{cases} \chi_i & p = \mathrm{DB}_i \\ p & p \neq \mathrm{DB}_i \end{cases}$



$\mathrm{sub}(\mathrm{w}_i, p) = \begin{cases} \chi_i^D & p = \mathrm{W}_i \\ p & p \neq \mathrm{W}_i \end{cases}$

$\mathrm{sub}(\mathrm{db}_i, p) = \begin{cases} \chi_i^D & p = \mathrm{B}_i \\ p & p \neq \mathrm{B}_i \end{cases}$

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$$At_{\geq i} \stackrel{\mathrm{def}}{=} \{\mathrm{DW}_j, \mathrm{DB}_j, \mathrm{W}_j, \mathrm{B}_j\}_{i \leq j \leq n}$$

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Result

$$\chi \stackrel{\text{def}}{=} (\mathrm{MW} \vee \mathrm{MB}) \wedge \neg(\mathrm{MW} \wedge \mathrm{MB}) \wedge$$

Exactly one urn is placed in the room

$$\bigwedge_{i \in Ag} \big(P_i(\mathrm{MW}) = P_i(\mathrm{MB})\big) \wedge$$

Each agent considers each urn equally likely

$$\bigwedge_{p \in At_{\geq 1}} \neg p$$

Noone has drawn or written anything

Proposition

For all $1 \leq j \leq i$, *let* $f_j \in \{\mathrm{dw}_j, \mathrm{db}_j\}$ *and* $g_j \in \{\mathrm{w}_j, \mathrm{b}_j\}$. *Then*

$$[(\cup_{i \in Ag} i)^*]\chi \Rightarrow$$
$$[\mathrm{dw}_1][g_1][\mathrm{dw}_2][g_2][f_3][g_3] \ldots [f_i][g_i]\big(P_k(\mathrm{MW}) > P_k(\mathrm{MB})\big)$$

If it is common knowledge of χ, and the first two balls to be drawn are white, then after subsequent draws, every agent will consider the majority white urn to be more likely.

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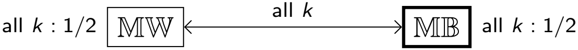
If it is common knowledge of χ, and the first two balls to be drawn are white, then after subsequent draws, every agent will consider the majority white urn to be more likely.

Let

- $\mathbb{MW}$ be a state whose only atom is MW and
- $\mathbb{MB}$ be a state whose only atom is MB

Any model satisfying $[(\cup_{k \in Ag} k)^*]\chi$ and MB is bisimilar to:



Recall

$$\chi \stackrel{\text{def}}{=} (\text{MW} \vee \text{MB}) \wedge \neg(\text{MW} \wedge \text{MB}) \wedge$$

Exactly one urn is placed in the room

$$\bigwedge_{k \in Ag} \left(P_k(\text{MW}) = P_k(\text{MB})\right) \wedge$$

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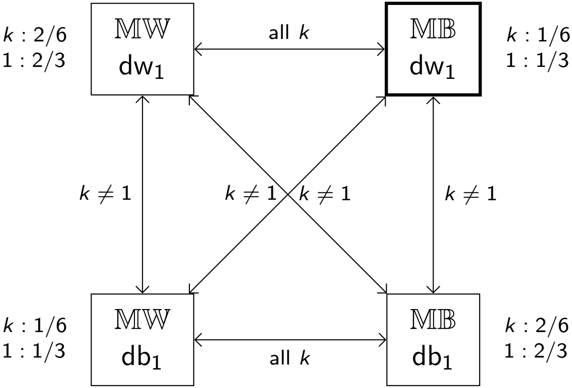
$$\bigwedge_{p \in At_{\geq 1}} \neg p$$

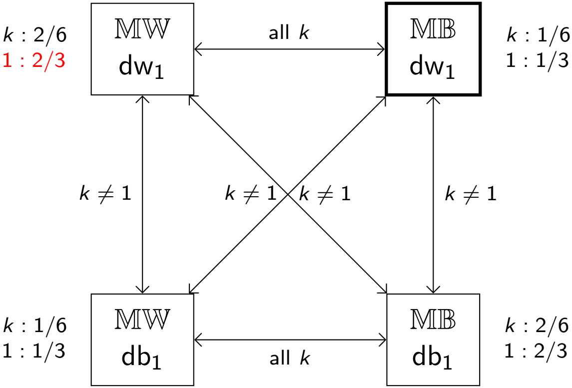
Noone has drawn or written anything

After agent 1 draws white



Agent 1 considers the majority white urn to be more likely, and will write white.

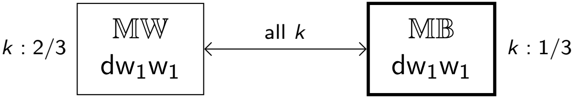
After agent 1 draws white



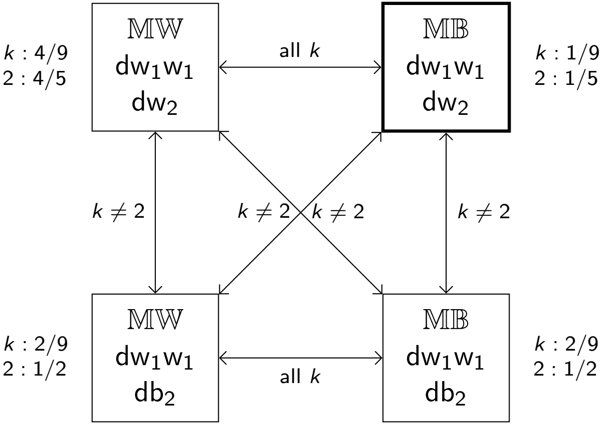
Agent 1 considers the majority white urn to be more likely, and will write white.

Agent 1 has just written her guess

The result of agent 1's guess is bisimilar (via generated submodel) to the following:

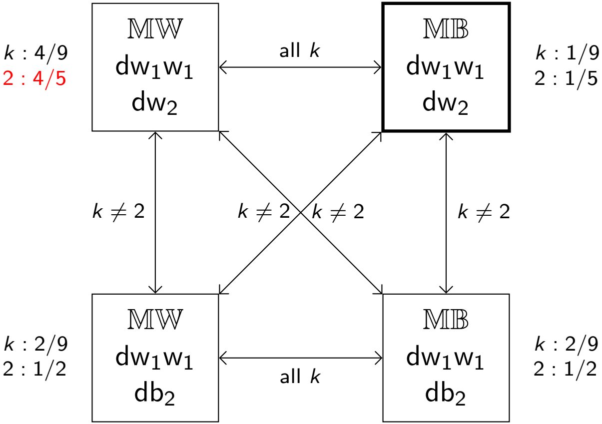


After second agent draws white



Agent 2 considers the majority white urn to be more likely, and will write white.

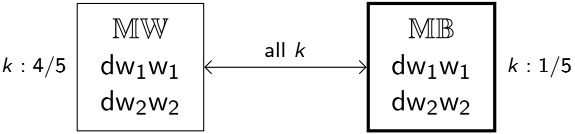
After second agent draws white



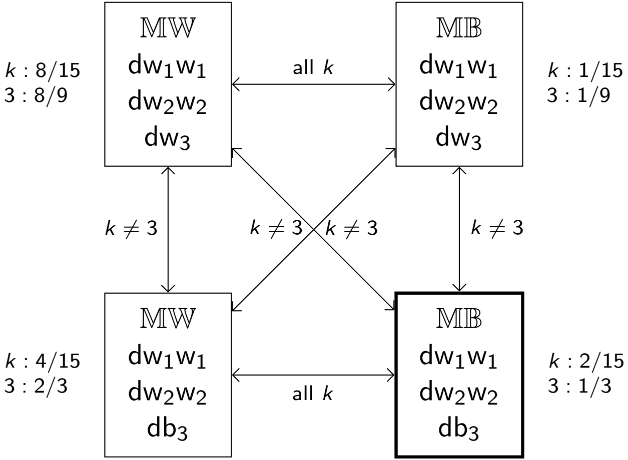
Agent 2 considers the majority white urn to be more likely, and will write white.

After agent 2 writes

The result of agent 2's guess is bisimilar (via generated submodel) to the following:

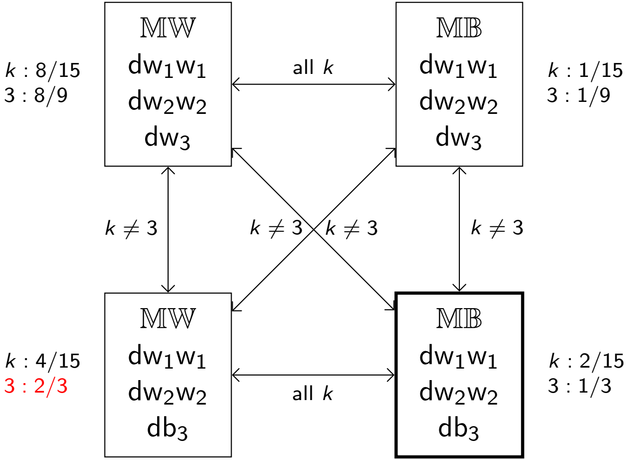


Result of agent 3 drawing black



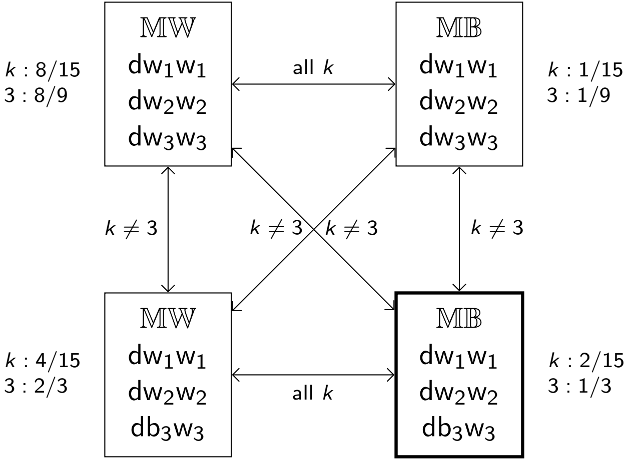
Agent 3 considers the majority white urn to be more likely, and will write white.

Result of agent 3 drawing black



Agent 3 considers the majority white urn to be more likely, and will write white.

Result of agent 3 writing

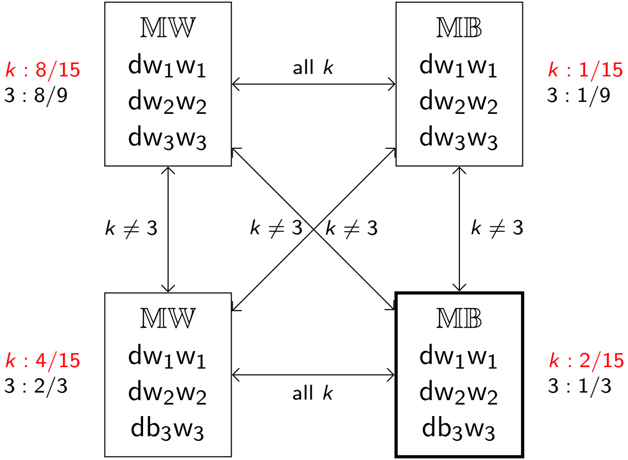


$\models P_k(MW) = 12/15 \wedge P_k(MB) = 3/15.$

$(\models P_k(MW) = 4/5 \wedge P_k(MB) = 1/5.)$

This is similar to the situation after agent 2 wrote.

Result of agent 3 writing



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A key observation about cascades

Higher order reasoning (even common knowledge) cannot necessarily prevent false cascades.

Conclusion

Logic can be used to help reason about very different subjects matters, ranging from physics, computer science, to social science.

- In quantum physics, logic can help us understand conditions that require quantum contextuality, making it easier to find experiments that support contextually
- In computation and concurrency, logic can help us reason about problems whether a program will "crash" or whether a concurrent system is "fair"
- In social science, logic can be used to reason formally about complex group behaviors, such as informational cascades.

Thank you!