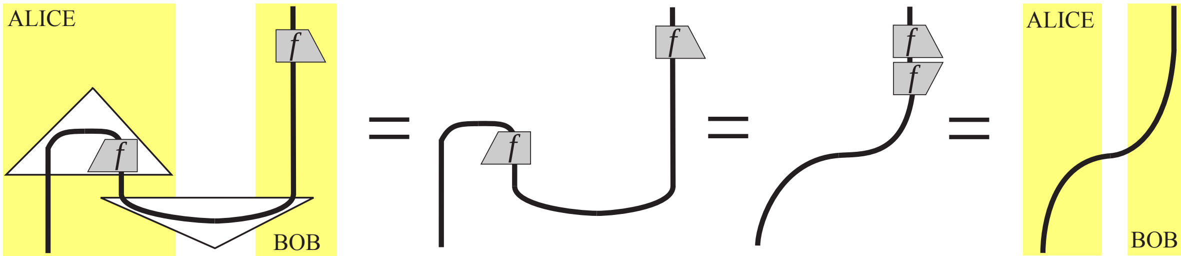
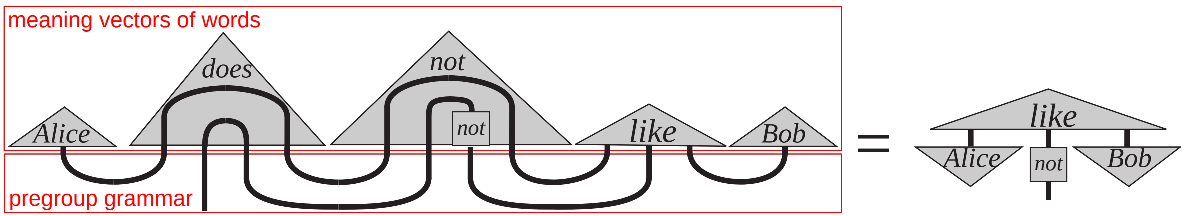
LOGIC OF QUANTUM MECHANICS – TAKE II

Bob Coecke — Oxford-CS-QG — arXiv:1204.3458

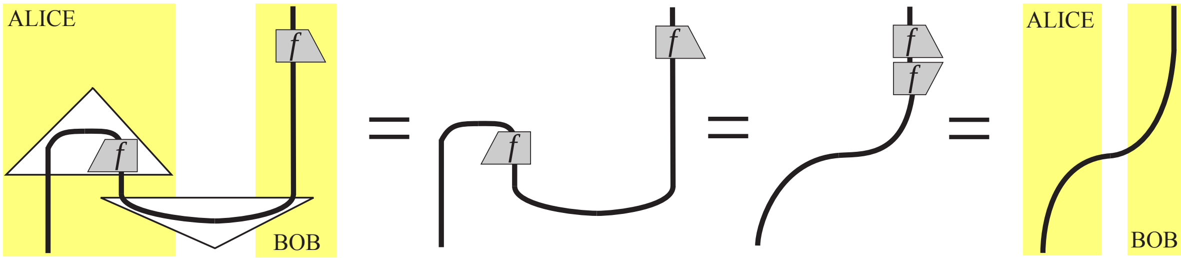


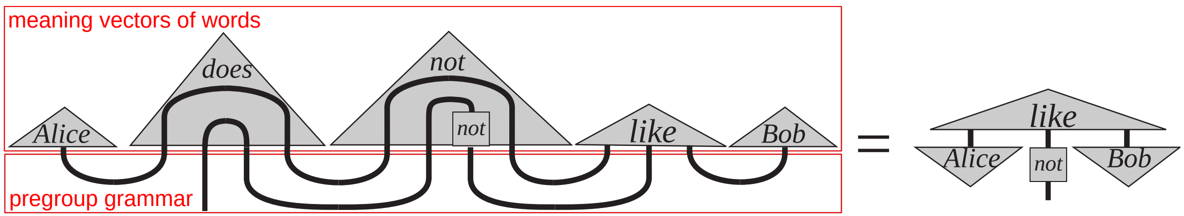


EPSRC
Engineering and Physical Sciences Research Council
Department of the Navy
ONR
Science & Technology
QICS
Information Society Technologies
CORDIS
Sixth Framework Programme
fq(x)
Foundational Questions Institute
John Templeton Foundation
The Leverhulme Trust
Air Force Office of Scientific Research
United States Air Force

LOGIG FOR COMPOSITION & INTERACTION

Bob Coecke — Oxford-CS-QG — arXiv:1204.3458

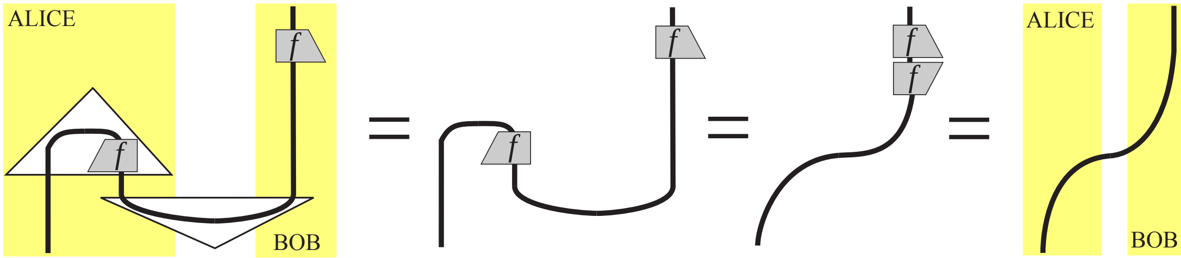


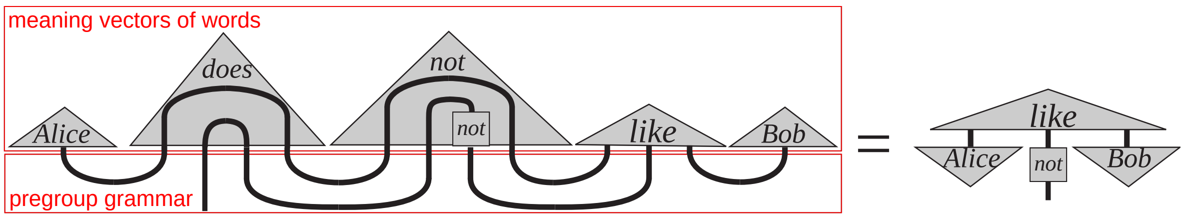


EPSRC
Engineering and Physical Sciences
Research Council
DEPARTMENT OF THE NAVY
ONR
Science & Technology
QICS
Information Society Technologies
CORDIS
Sixth Framework Programme
fq(x)
Foundational Questions Institute
John
Templeton
Foundation
The Leverhulme Trust
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
UNITED STATES AIR FORCE

VS. LOGIC FOR ISOLATION & REDUCTION

Bob Coecke — Oxford-CS-QG — arXiv:1204.3458





EPSRC Engineering and Physical Sciences Research Council
Department of the Navy ONR Science & Technology
QICS
Information Society Technologies
CORDIS
Sixth Framework Programme
fq(x) Foundational Questions Institute
John Templeton Foundation
The Leverhulme Trust
Air Force Office of Scientific Research United States Air Force

- **New foundational insights:**
  -
  -

- **New foundational insights:**
  - –
  - –
- **Drives new technologies:**
  - –
  - –

- **New foundational insights:**
  - Quantumness is all about **composition**
  -
- **Drives new technologies:**
  -
  -

- **New foundational insights:**
  - Quantumness is all about **composition**
  - Grammar is all about **meaning-flow**
- **Drives new technologies:**
  -
  -

- **New foundational insights:**
  - Quantumness is all about **composition**
  - Grammar is all about **meaning-flow**
- **Drives new technologies:**
  - 1st high-level account on quantum technologies
  -

- **New foundational insights:**
  - Quantumness is all about **composition**
  - Grammar is all about **meaning-flow**
- **Drives new technologies:**
  - 1st high-level account on quantum technologies
  - 1st compositional distributional meaning model

— *genesis* —

— ***genesis*** —

[von Neumann 1932] Formalized quantum mechanics in "Mathematische Grundlagen der Quantenmechanik"

— *genesis* —

[von Neumann 1932] Formalized quantum mechanics in "Mathematische Grundlagen der Quantenmechanik"

[von Neumann to Birkhoff 1935] "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space no more." (sic)

— ***genesis*** —

[von Neumann 1932] Formalized quantum mechanics in "Mathematische Grundlagen der Quantenmechanik"

[von Neumann to Birkhoff 1935] "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space no more." (sic)

[Birkhoff & von Neumann 1936] *The Logic of Quantum Mechanics* in *Annals of Mathematics*.

— *genesis* —

[von Neumann 1932] Formalized quantum mechanics in "Mathematische Grundlagen der Quantenmechanik"

[von Neumann to Birkhoff 1935] "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space no more." (sic)

[Birkhoff & von Neumann 1936] *The Logic of Quantum Mechanics* in *Annals of Mathematics*.

[1936 – ...] many followed them, ...

— *genesis* —

[von Neumann 1932] Formalized quantum mechanics in "Mathematische Grundlagen der Quantenmechanik"

[von Neumann to Birkhoff 1935] "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space no more." (sic)

[Birkhoff & von Neumann 1936] *The Logic of Quantum Mechanics* in *Annals of Mathematics*.

[1936 – ...] many followed them, ... and **FAILED**.

— *mathematics* —

— *mathematics* —

Hilbert space stuff: continuum, field structure of complex numbers, vector space over it, inner-product, etc.

— *mathematics* —

Hilbert space stuff: continuum, field structure of complex numbers, vector space over it, inner-product, etc.

WHY?

— *mathematics* —

Hilbert space stuff: continuum, field structure of complex numbers, vector space over it, inner-product, etc.

WHY?

von Neumann: only used *it* since *it* was ‘available’.

— *physics* —

Schrödinger (1935):

— *physics* —

Schrödinger (1935): the stuff which is the true soul of quantum theory is **how quantum systems compose**.

— *physics* —

Schrödinger (1935): the stuff which is the true soul of quantum theory is **how quantum systems compose**.

Last 20 year discoveries: Schrödinger was right!

— *the game plan* —

— *the game plan* —

Task 0. Solve:

$$\frac{\texttt{tensor product structure}}{\texttt{the other stuff}} = ???$$

— *the game plan* —

Task 0. Solve:

$$\frac{\texttt{tensor product structure}}{\texttt{the other stuff}} = ???$$

Task 1. Investigate which assumptions (i.e. structure) is needed to deduce **physical phenomena**.

— *the game plan* —

Task 0. Solve:

$$\frac{\texttt{tensor product structure}}{\texttt{the other stuff}} = ???$$

Task 1. Investigate which assumptions (i.e. structure) is needed to deduce **physical phenomena**.

Task 2. Do we encounter the resulting “interaction structure” elsewhere in **our classical reality**.

$$\frac{\texttt{tensor product structure}}{\texttt{the other stuff}} = {\color{red}???}$$

1. Let A be a raw potato.

1. Let A be a raw potato.

A admits many *states* e.g. dirty, clean, skinned, ...

1. Let A be a raw potato.

A admits many *states* e.g. dirty, clean, skinned, ...

2. We want to *process* A into cooked potato B.

B admits many *states* e.g. boiled, fried, deep fried, baked with skin, baked without skin, ...

1. Let A be a raw potato.

A admits many *states* e.g. dirty, clean, skinned, ...

2. We want to *process* A into cooked potato B.

B admits many *states* e.g. boiled, fried, deep fried, baked with skin, baked without skin, ... Let

$$A \xrightarrow{f} B \qquad A \xrightarrow{f'} B \qquad A \xrightarrow{f''} B$$

be boiling, frying, baking.

1. Let A be a raw potato.

A admits many *states* e.g. dirty, clean, skinned, ...

2. We want to *process* A into cooked potato B.

B admits many *states* e.g. boiled, fried, deep fried, baked with skin, baked without skin, ... Let

$$A \xrightarrow{f} B \qquad A \xrightarrow{f'} B \qquad A \xrightarrow{f''} B$$

be boiling, frying, baking. *States* are *processes*

$$\mathrm{I} := \text{unspecified} \xrightarrow{\psi} A.$$

3. Let

$$A \xrightarrow{g \circ f} C$$

be the *composite process* of first boiling $A \xrightarrow{f} B$ and then salting $B \xrightarrow{g} C$.

3. Let

$$A \xrightarrow{g \circ f} C$$

be the *composite process* of first boiling $A \xrightarrow{f} B$ and then salting $B \xrightarrow{g} C$. Let

$$X \xrightarrow{\mathbf{1}_X} X$$

be doing nothing. We have $\mathbf{1}_Y \circ \xi = \xi \circ \mathbf{1}_X = \xi$.

4. Let $A \otimes D$ be potato A and carrot D

4. Let $A \otimes D$ be potato A and carrot D and let

$$A \otimes D \xrightarrow{f \otimes h} B \otimes E$$

be boiling potato while frying carrot.

4. Let $A \otimes D$ be potato A and carrot D and let

$$A \otimes D \xrightarrow{f \otimes h} B \otimes E$$

be boiling potato while frying carrot. Let

$$C \otimes F \xrightarrow{x} M$$

be mashing spice-cook-potato and spice-cook-carrot.

5. Total *process*:

$$A \otimes D \xrightarrow{f \otimes h} B \otimes E \xrightarrow{g \otimes k} C \otimes F \xrightarrow{x} M = A \otimes D \xrightarrow{x \circ (g \otimes k) \circ (f \otimes h)} M.$$

5. Total *process*:

$$A \otimes D \xrightarrow{f \otimes h} B \otimes E \xrightarrow{g \otimes k} C \otimes F \xrightarrow{x} M = A \otimes D \xrightarrow{x \circ (g \otimes k) \circ (f \otimes h)} M.$$

6. *Recipe* = *composition structure* on *processes*.

5. Total *process*:

$$A \otimes D \xrightarrow{f\otimes h} B \otimes E \xrightarrow{g\otimes k} C \otimes F \xrightarrow{x} M = A \otimes D \xrightarrow{x\circ(g\otimes k)\circ(f\otimes h)} M.$$

6. *Recipe* = *composition structure* on *processes*.

7. *Laws governing recipes*:

$$(\mathbf{1}_B \otimes g) \circ (f \otimes \mathbf{1}_C) = (f \otimes \mathbf{1}_D) \circ (\mathbf{1}_A \otimes g)$$

5. Total *process*:

$$A \otimes D \xrightarrow{f\otimes h} B \otimes E \xrightarrow{g\otimes k} C \otimes F \xrightarrow{x} M = A \otimes D \xrightarrow{x\circ(g\otimes k)\circ(f\otimes h)} M.$$

6. *Recipe* = *composition structure* on *processes*.

7. *Laws governing recipes*:

$$(\mathbf{1}_B \otimes g) \circ (f \otimes \mathbf{1}_C) = (f \otimes \mathbf{1}_D) \circ (\mathbf{1}_A \otimes g)$$

i.e.

boil potato then fry carrot = fry carrot then boil potato

5. Total *process*:

$$A \otimes D \xrightarrow{f \otimes h} B \otimes E \xrightarrow{g \otimes k} C \otimes F \xrightarrow{x} M = A \otimes D \xrightarrow{x \circ (g \otimes k) \circ (f \otimes h)} M.$$

6. *Recipe* = *composition structure* on *processes*.

7. *Laws governing recipes*:

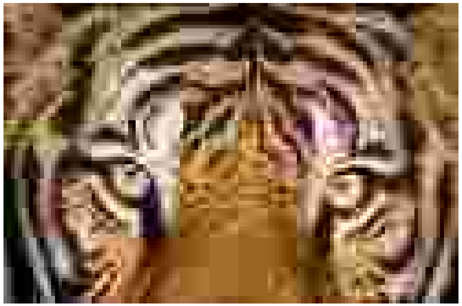
$$(\mathbf{1}_B \otimes g) \circ (f \otimes \mathbf{1}_C) = (f \otimes \mathbf{1}_D) \circ (\mathbf{1}_A \otimes g)$$

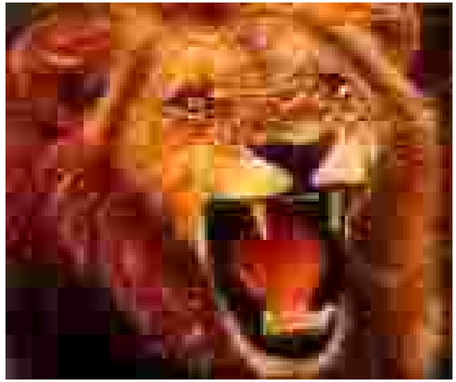
i.e.

boil potato then fry carrot = fry carrot then boil potato

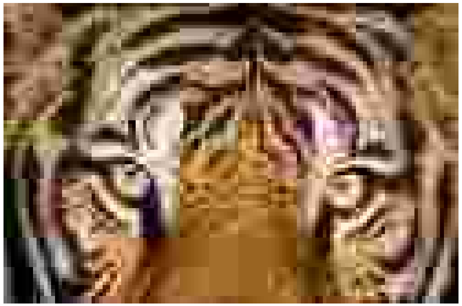
$\Rightarrow$ **Symmetric Monoidal Category**

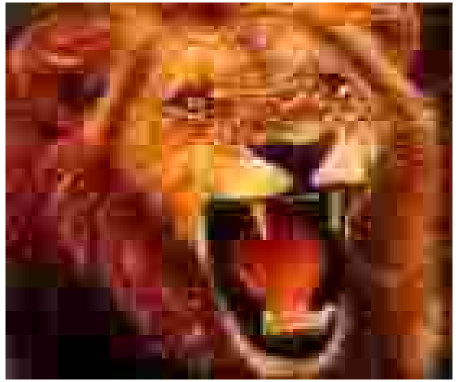
— *Why does a tiger have stripes and a lion doesn't?* —





— ***Why does a tiger have stripes and a lion doesn't?*** —





$$\text{prey} \otimes \text{predator} \otimes \text{environment}$$
$$\Big\downarrow \text{hunt}$$
$$\text{dead prey} \otimes \text{eating predator}$$

AN ALTERNATIVE TO REDUCTIONISM

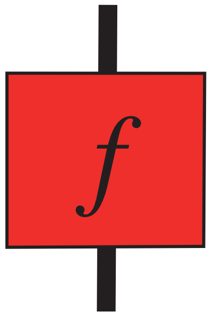
AND IT GETS EVEN BETTER

BOXES AND WIRES

Roger Penrose (1971) *Applications of negative dimensional tensors*. In: Combinatorial Mathematics and its Applications, 221–244. Academic Press.

André Joyal and Ross Street (1991) *The Geometry of tensor calculus* I. Advances in Mathematics **88**, 55–112.

— ***wire and box language*** —

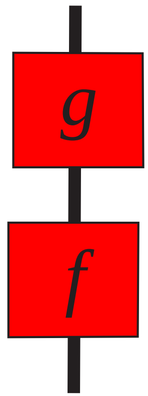


wire := **system** ; box := **process**

— *composing boxes* —

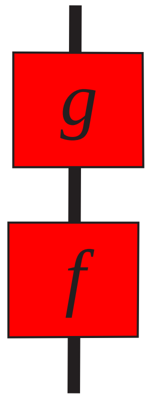
— ***composing boxes*** —

sequential composition:

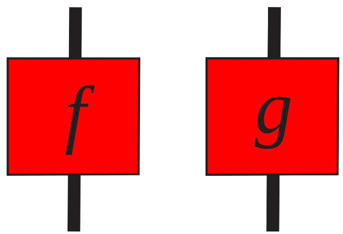
$g \circ f \equiv$ 

— *composing boxes* —

sequential composition:

$g \circ f \equiv$ 

parallel composition:

$f \otimes g \equiv$ 

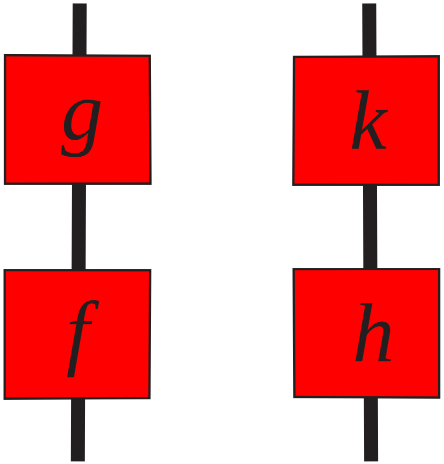
— *merely a new notation?* —

— ***merely a new notation?*** —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$

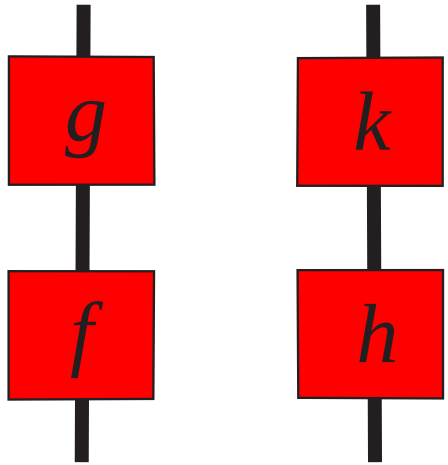
— merely a new notation? —

$$(g \circ f) \otimes (k \circ h)$$



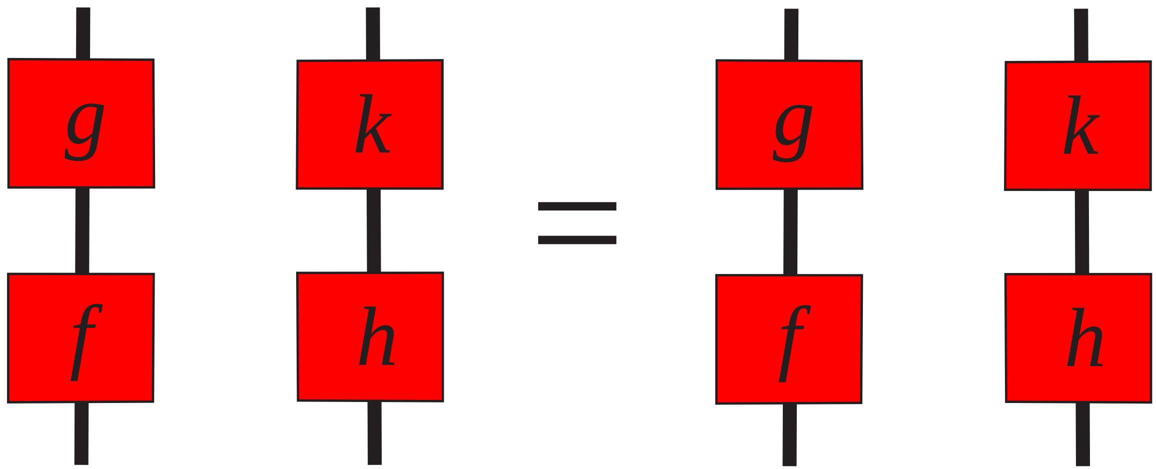
— *merely a new notation?* —

$$(g \otimes k) \circ (f \otimes h)$$



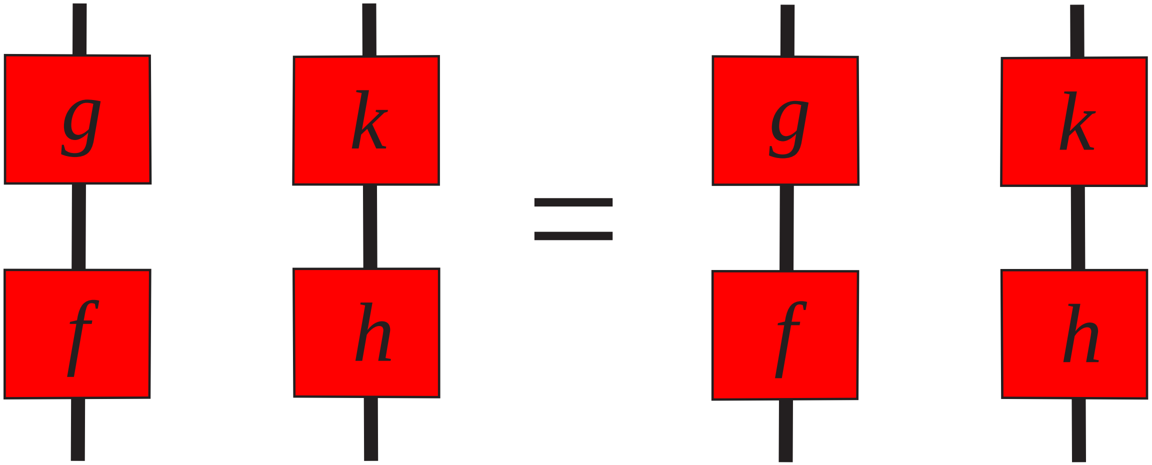
— *merely a new notation?* —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$



— *merely a new notation?* —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$

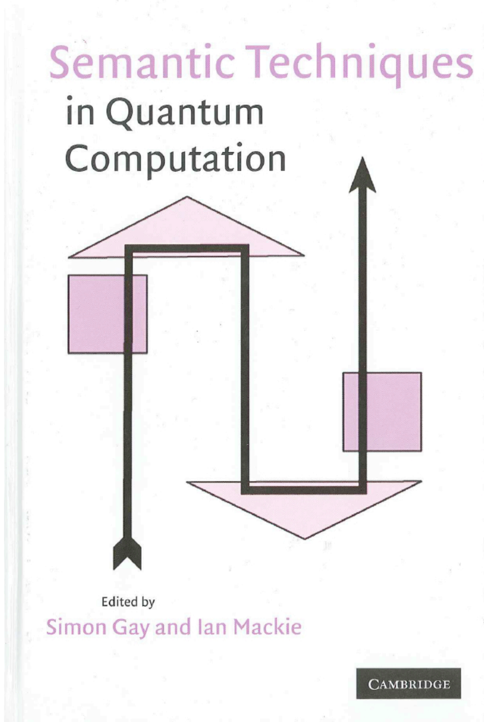


peel potato and then fry it,
while,
clean carrot and then boil it

=

peel potato while clean carrot,
and then,
fry potato while boil carrot

QUANTUM PROCESSES



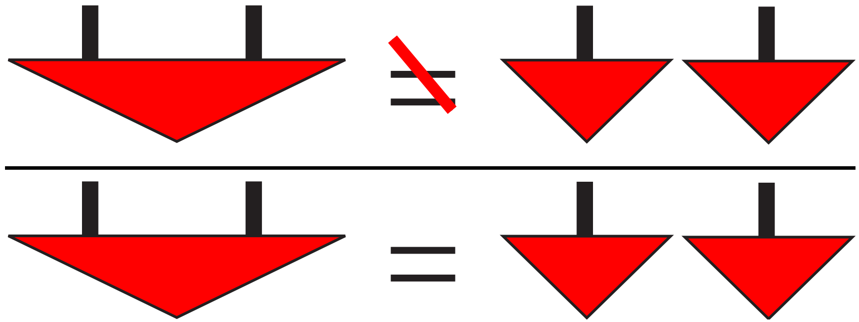
BC (2003) *The logic of entanglement. An invitation.* `quant-ph/0402014`

Samson Abramsky & BC (2004) *A categorical semantics for quantum protocols*. In: LiCS'04. `quant-ph/0402130`

— *asserting “verschränkung”* —

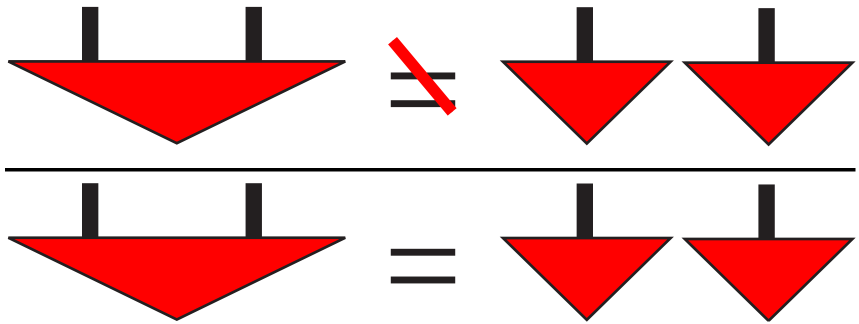
— *asserting “verschränkung”* —

$$\frac{\textit{quantum}}{\textit{classical}} =$$

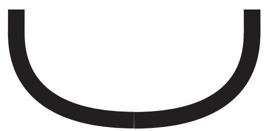


— *asserting "verschränkung"* —

$\frac{\textit{quantum}}{\textit{classical}} =$

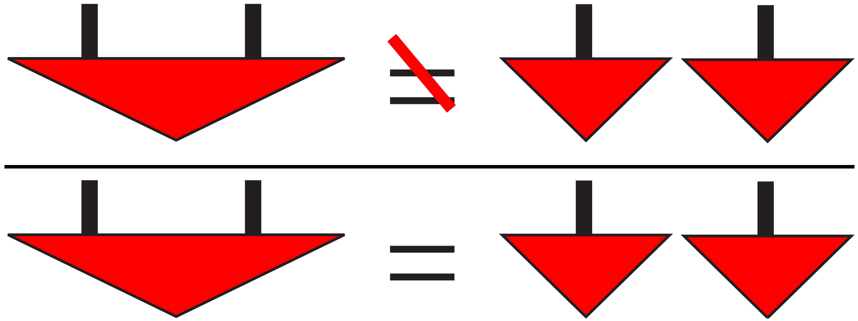


$\Rightarrow$ introduce 'parallel wire' between systems:

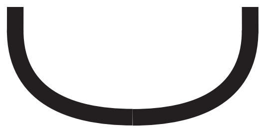


— *asserting "verschränkung"* —

$$\frac{\textit{quantum}}{\textit{classical}} =$$



⇒ introduce 'parallel wire' between systems:

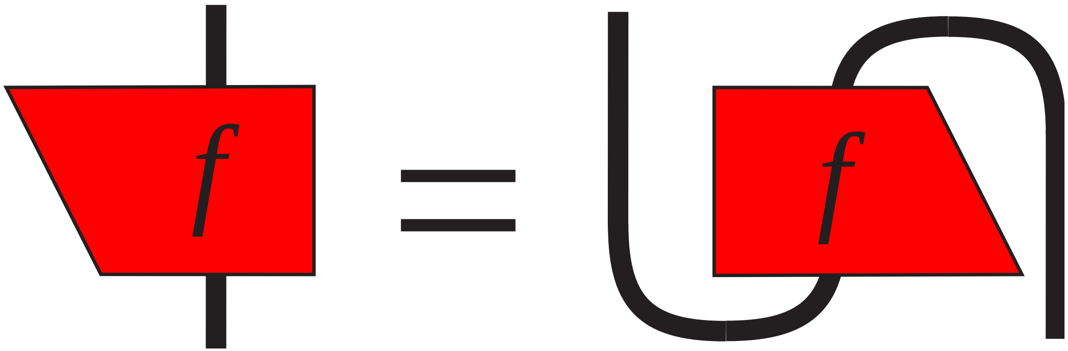


subject to: **only topology matters!**

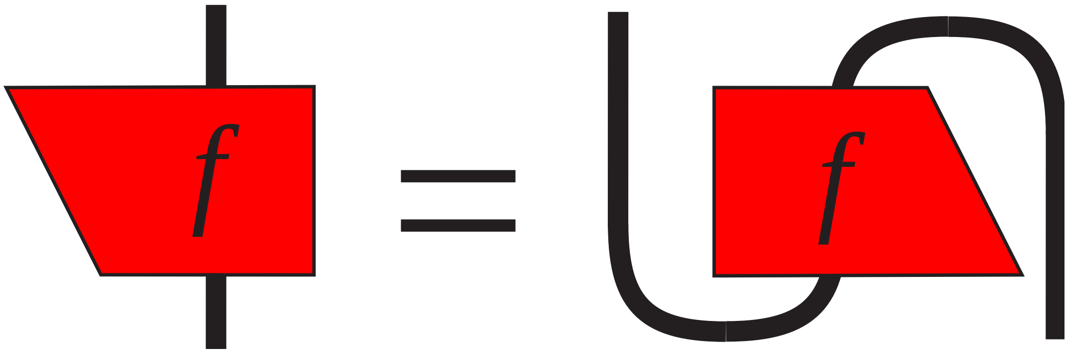
— *asserting “verschränkung”* —

E.g.

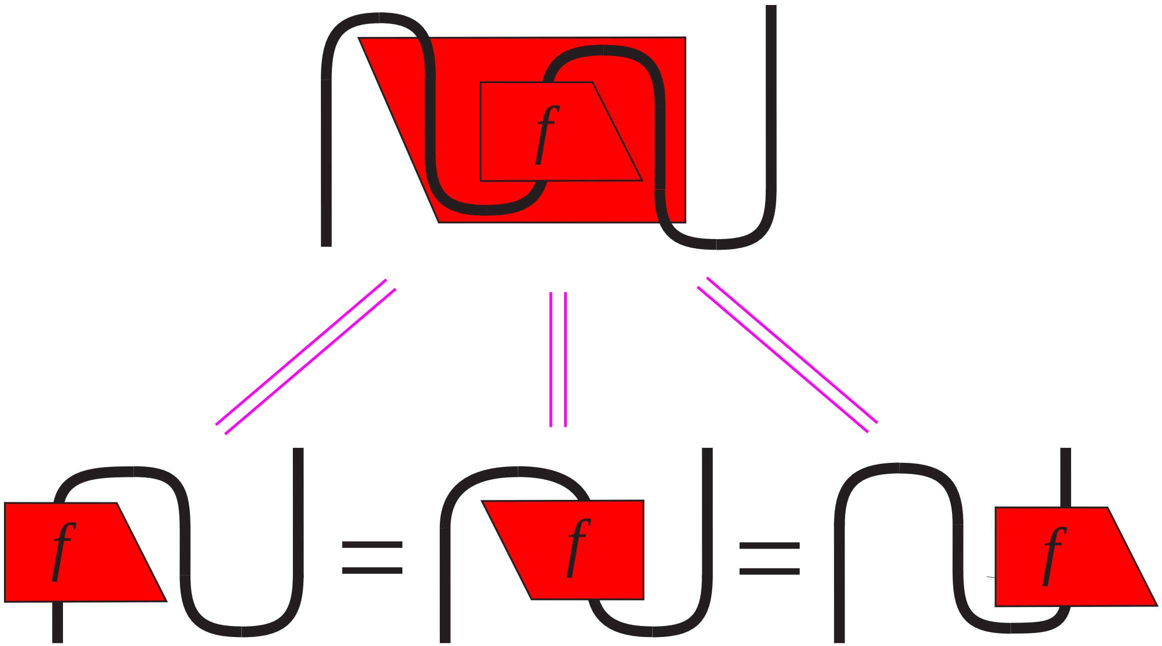
— *the transpose* —



— *the transpose* —

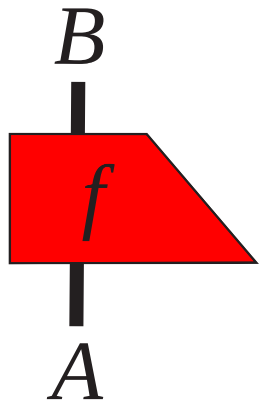


— *sliding* —

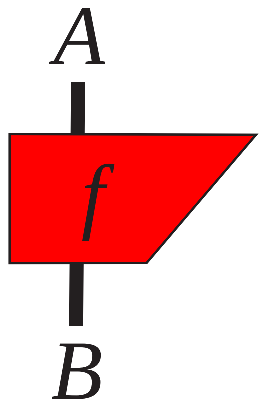


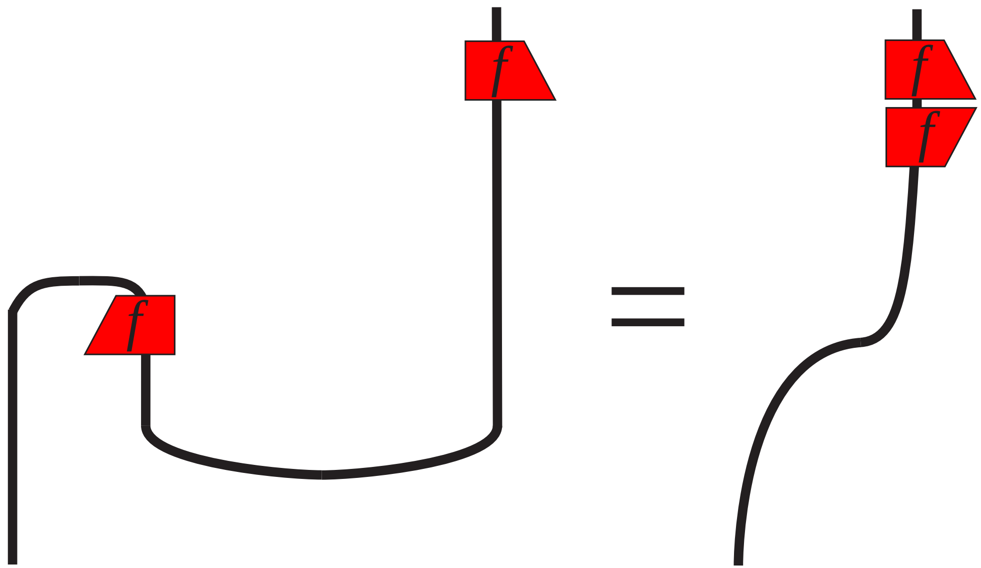
— *state-question duality* —

— *state-question duality* —

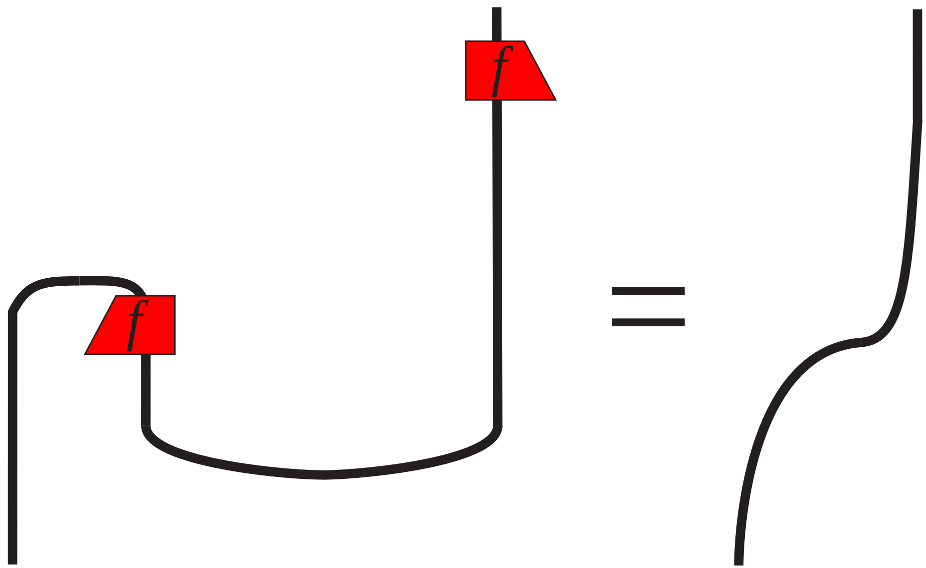


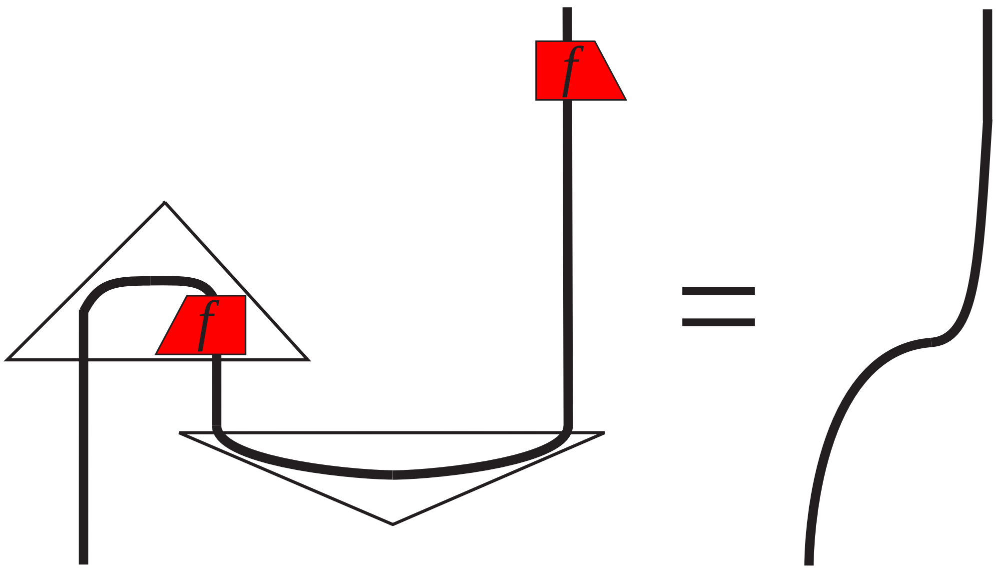
— *state-question duality* —

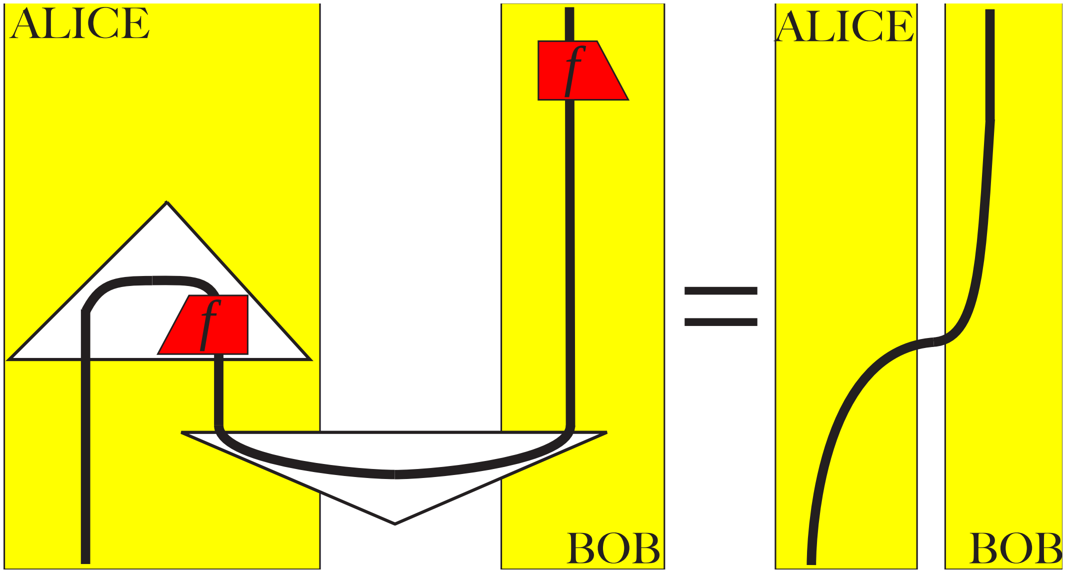




f
f
=
f
f


f
f


f
f
=



⇒ quantum teleportation

— *completeness* —

Theorem. [Kelly & Laplaza 1980; Selinger 2005; Hasegawa, Hofmann & Plotkin 2007; Selinger 2008]

TFAE:

— *completeness* —

Theorem. [Kelly & Laplaza 1980; Selinger 2005; Hasegawa, Hofmann & Plotkin 2007; Selinger 2008]

TFAE:

- An equational statement holds between **diagrams**;

— *completeness* —

Theorem. [Kelly & Laplaza 1980; Selinger 2005; Hasegawa, Hofmann & Plotkin 2007; Selinger 2008]

TFAE:

- An equational statement holds between **diagrams**;
- It holds in **dagger compact categories**;

— *completeness* —

Theorem. [Kelly & Laplaza 1980; Selinger 2005; Hasegawa, Hofmann & Plotkin 2007; Selinger 2008]

TFAE:

- An equational statement holds between **diagrams**;
- It holds in **dagger compact categories**;
- It holds for the dagger compact category of **Hilbert spaces, linear maps, tensor product and adjoint**.

— *full expressivity* —

— *full expressivity* —

Quantum Computer Science course:

- quantum computational models,
- quantum cryptography,
- quantum non-locality,
- quantum information,
- quantum algorithms, . . .

— *full expressivity* —

Quantum Computer Science course:

- quantum computational models,
- quantum cryptography,
- quantum non-locality,
- quantum information,
- quantum algorithms, . . .

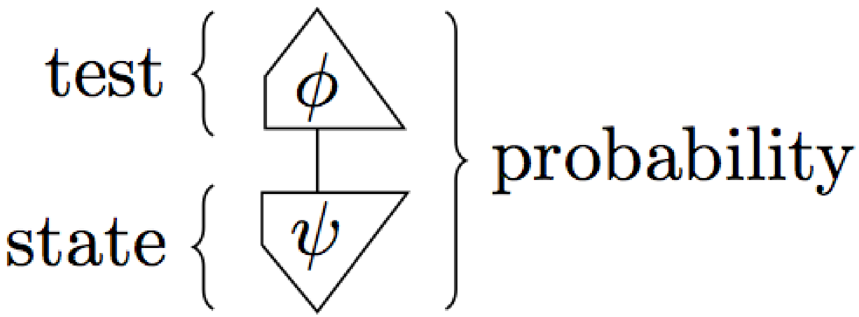
Forthcoming textbook:

BC & Aleks Kissinger, *Picturing Quantum Processes*.
Cambridge University Press, forthcoming.

— *Born-rule and mixing* —

BC (2005) De-linearizing linearity. arXiv:quant-ph/0506134

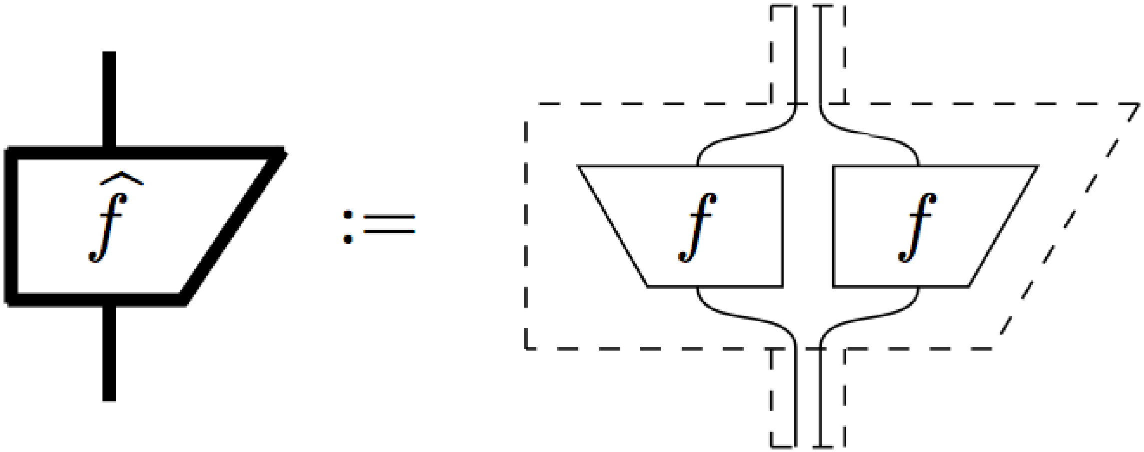
Peter Selinger (2005) DCCCs & CPMs



— *Born-rule and mixing* —

BC (2005) De-linearizing linearity. arXiv:quant-ph/0506134

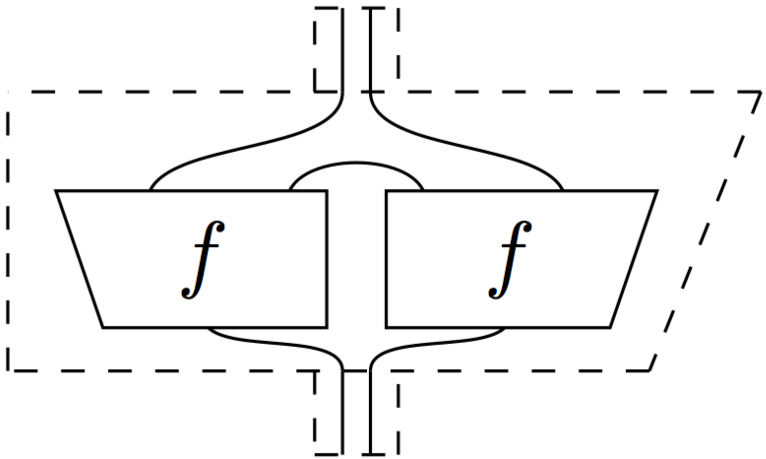
Peter Selinger (2005) DCCCs & CPMs



— *Born-rule and mixing* —

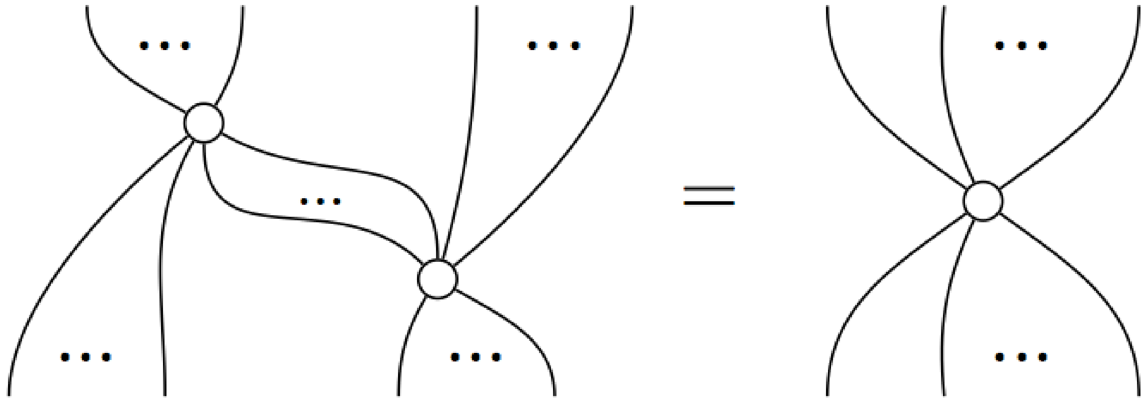
BC (2005) De-linearizing linearity. arXiv:quant-ph/0506134

Peter Selinger (2005) DCCCs & CPMs



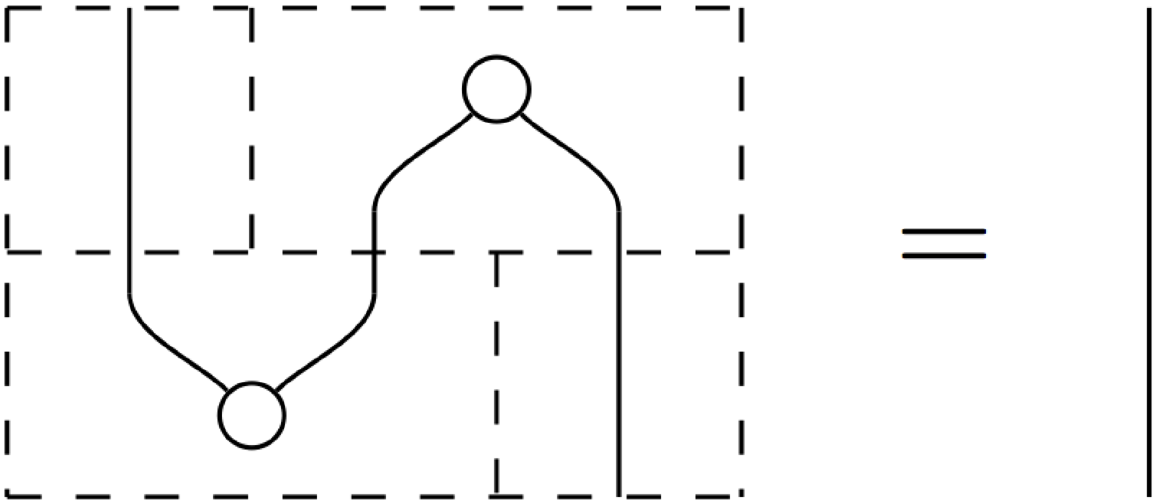
— *classical data and classical maps* —

BC & Dusko Pavlovic (2006) Quantum measurements without sums. arXiv:quant-ph/0608035; BC, Eric O. Paquette and DP (2009) Classical and quantum structuralism. arXiv:0904.1997.



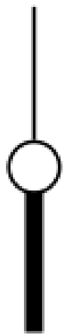
— *classical data and classical maps* —

BC & Dusko Pavlovic (2006) Quantum measurements without sums. arXiv:quant-ph/0608035; BC, Eric O. Paquette and DP (2009) Classical and quantum structuralism. arXiv:0904.1997.

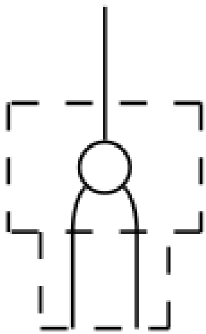


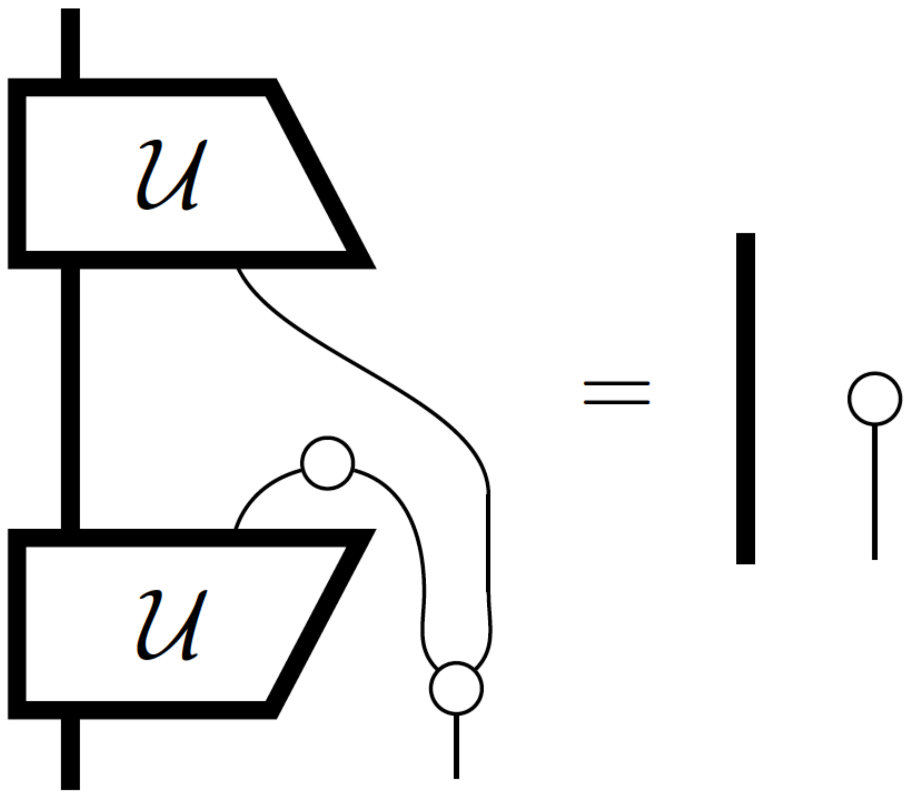
— *classical data and classical maps* —

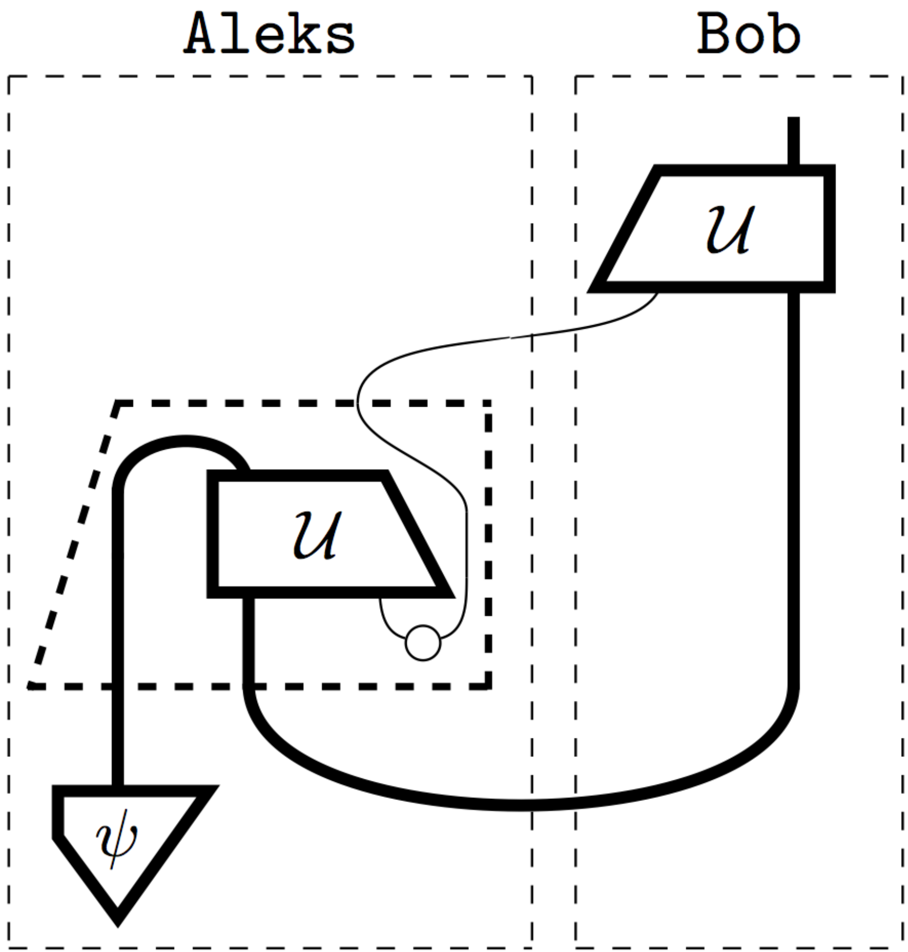
BC & Dusko Pavlovic (2006) Quantum measurements without sums. arXiv:quant-ph/0608035; BC, Eric O. Paquette and DP (2009) Classical and quantum structuralism. arXiv:0904.1997.



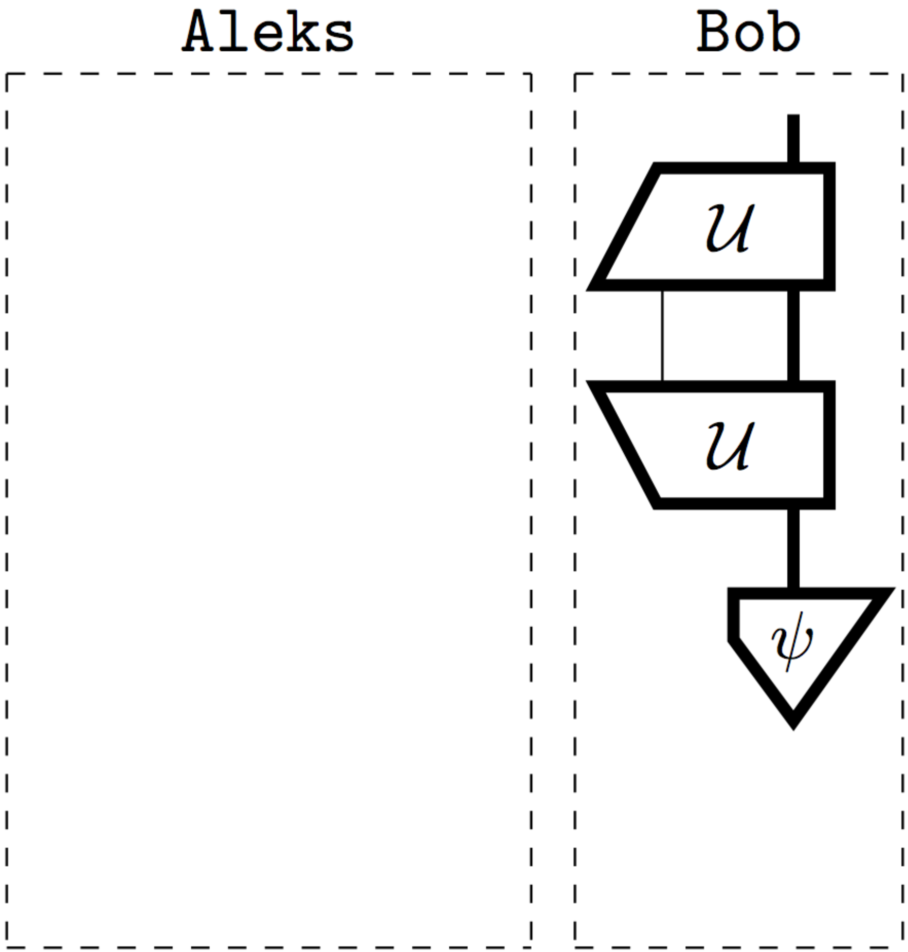
$:=$

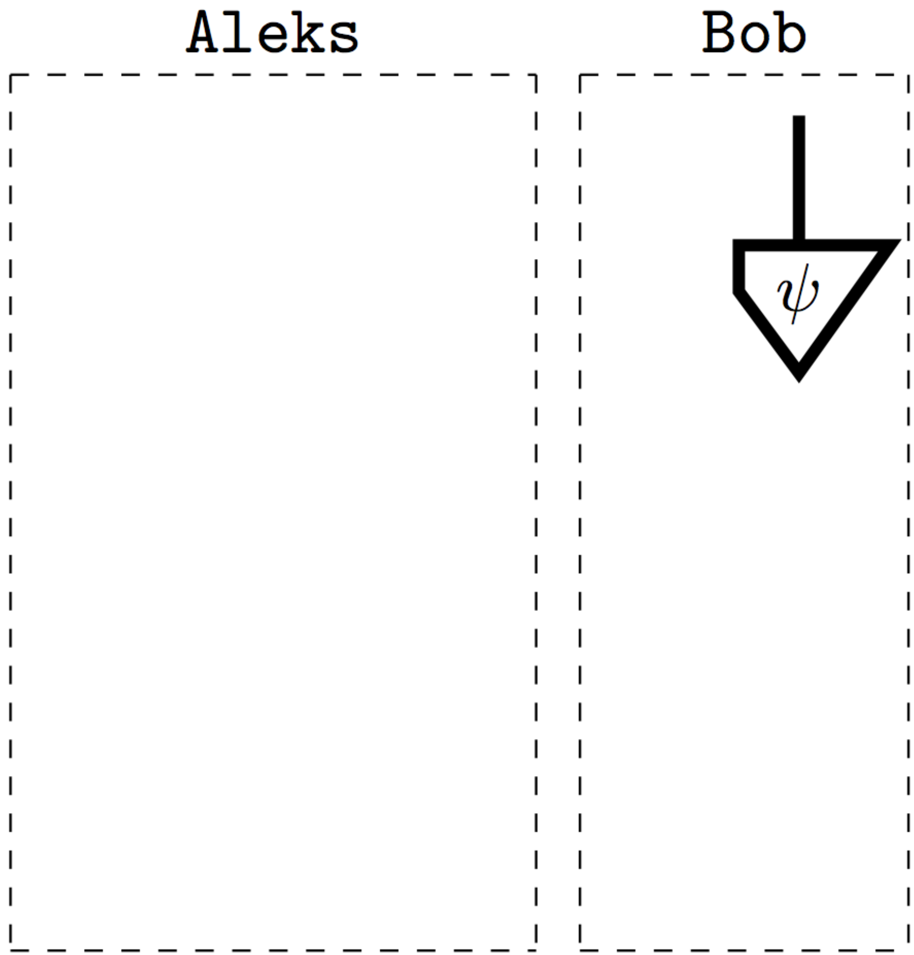



$\mathcal{U}$
$\mathcal{U}$
=



Aleks
Bob
$\mathcal{U}$
$\mathcal{U}$
ψ

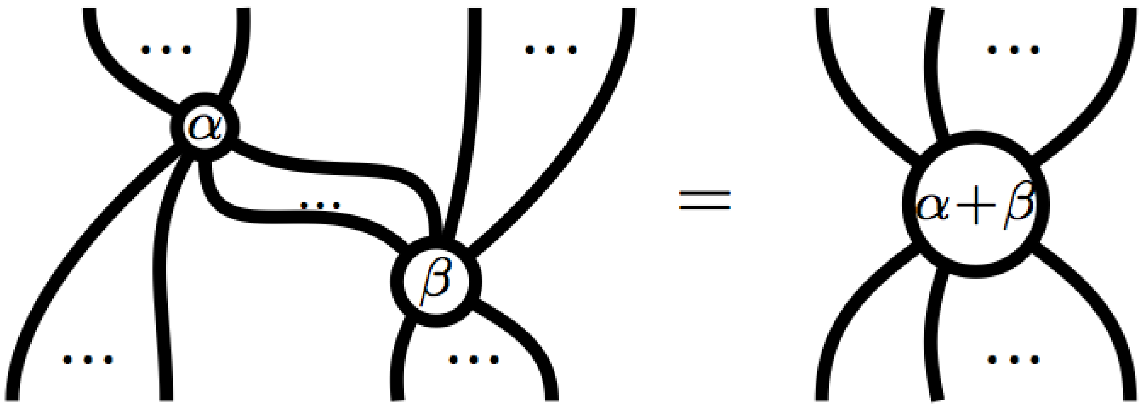

Aleks
Bob
$\mathcal{U}$
$\mathcal{U}$
ψ



Aleks
Bob
ψ

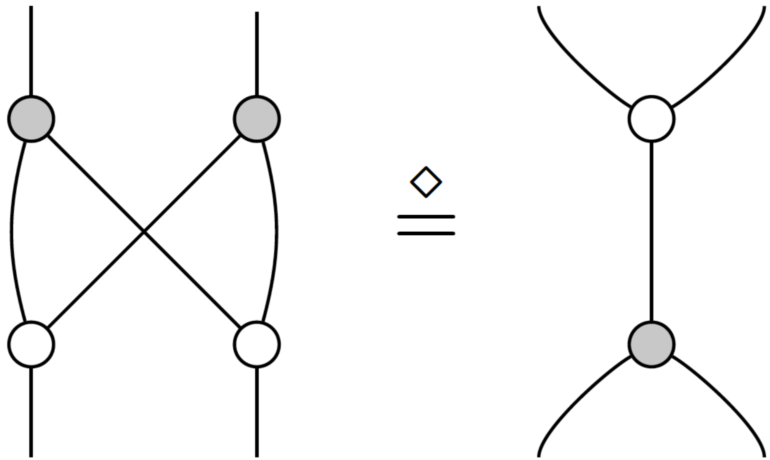
— *phases and complementarity: full linearity* —

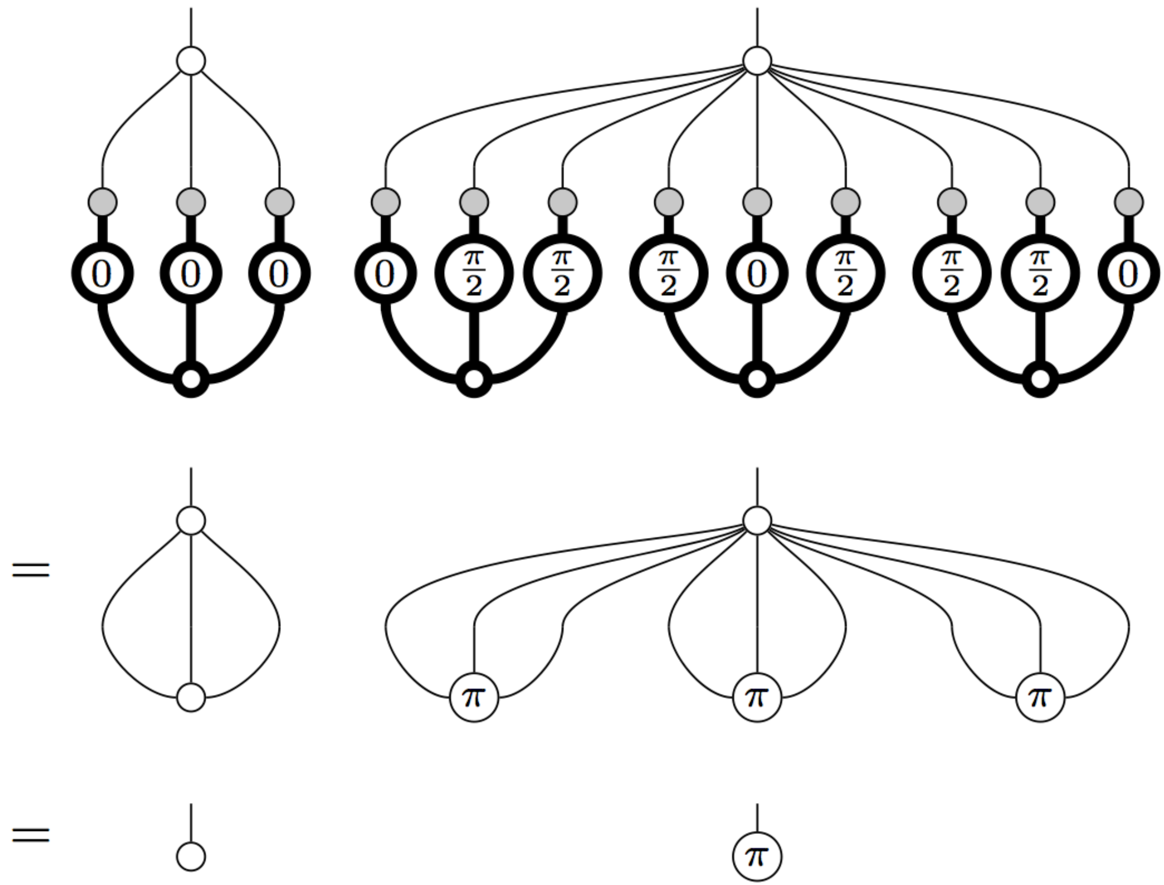
BC & Ross Duncan (2008, 2010) Interacting quantum observables (ICALP & NJP). arXiv:0906.4725

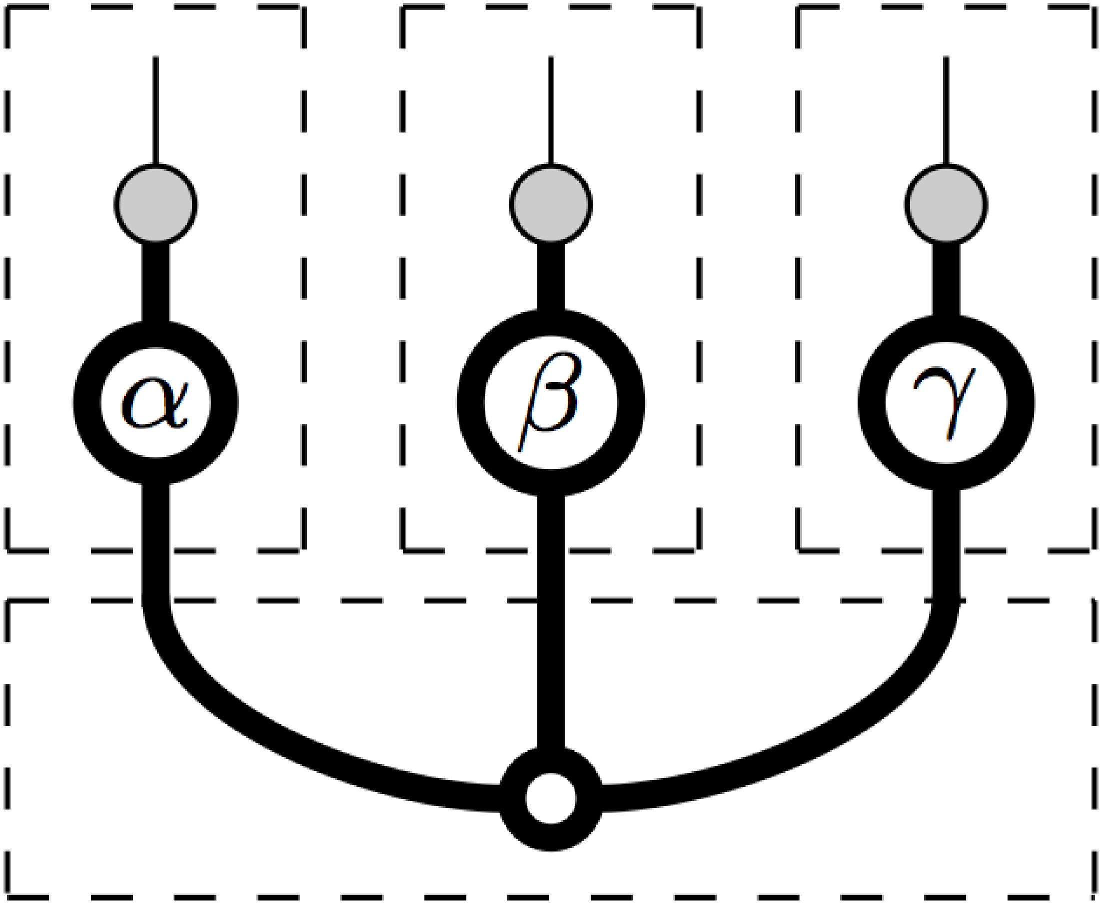


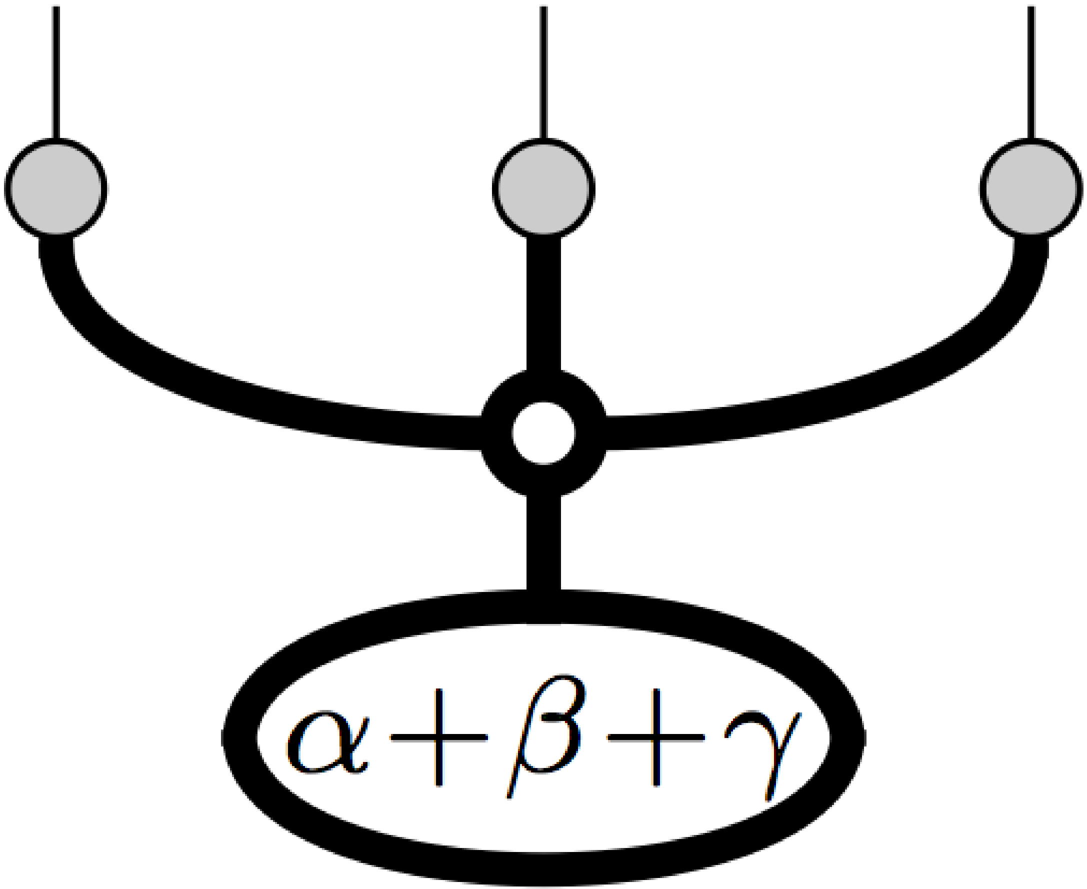
— *phases and complementarity: full linearity* —

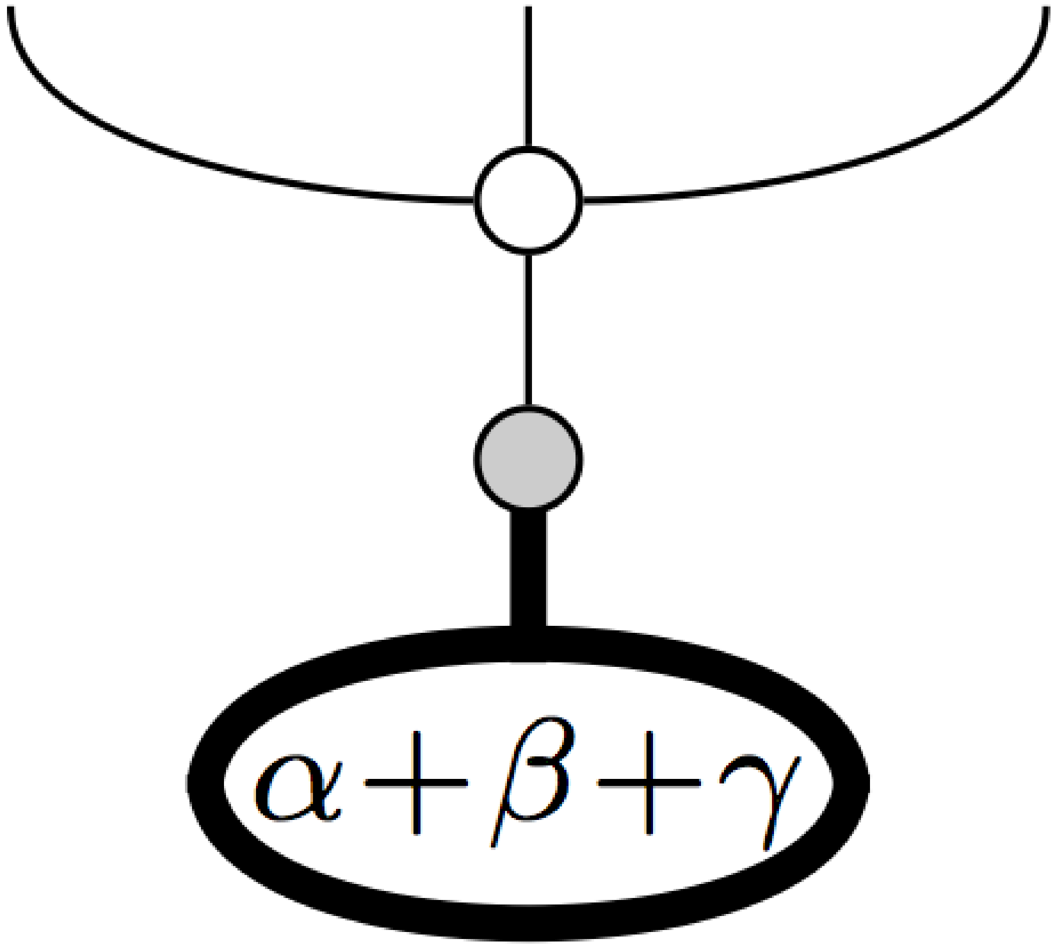
BC & Ross Duncan (2008, 2010) Interacting quantum observables (ICALP & NJP). arXiv:0906.4725

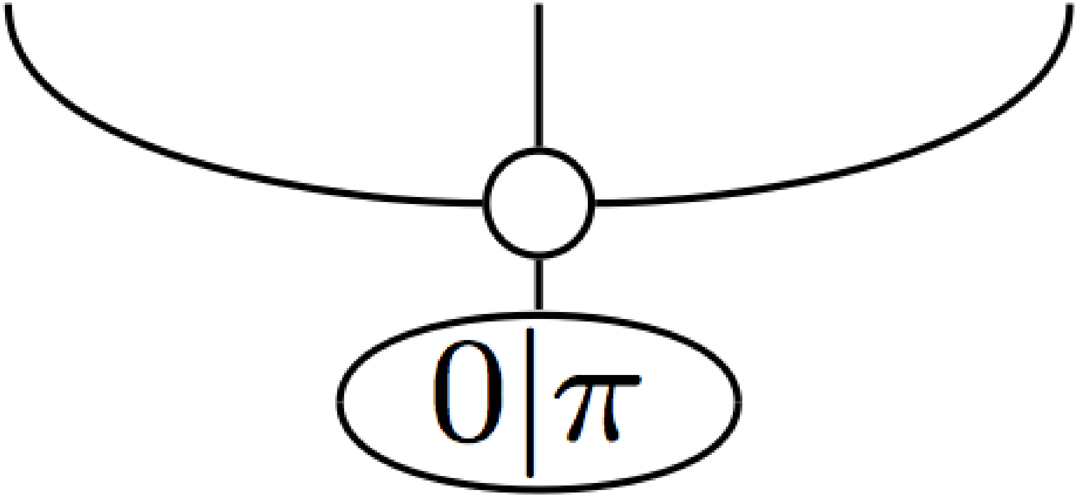


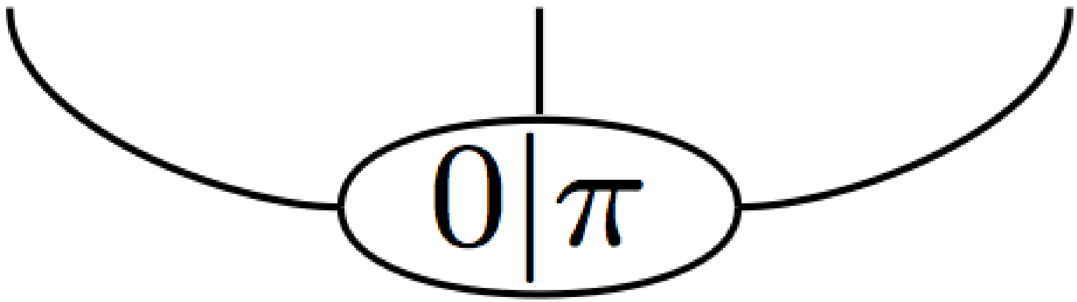



α
β
γ


$\alpha+\beta+\gamma$


$\alpha+\beta+\gamma$


0|π


0|π

— *completeness II* —

Not a complete surprise but non-trivial:

Miriam Backens (2013) The ZX-calculus is complete for stabilizer quantum mechanics. arXiv:1307.7025

— *completeness II* —

Not a complete surprise but non-trivial:

Miriam Backens (2013) The ZX-calculus is complete for stabilizer quantum mechanics. arXiv:1307.7025

Maybe more of a surprise:

Miriam Backens (2014) The ZX-calculus is approximately complete for single qubits.

— *experiment: kindergarten quantum mechanics* —

Contest in problem solving between:

- Children using quantum picturalism
- Physics teachers using ordinary QM

From: BC (2010) ***Quantum picturalism***. Contemporary Physics **51**, 59–83.

— *experiment: kindergarten quantum mechanics* —

Contest in problem solving between:

- Children using quantum picturalism
- Physics teachers using ordinary QM

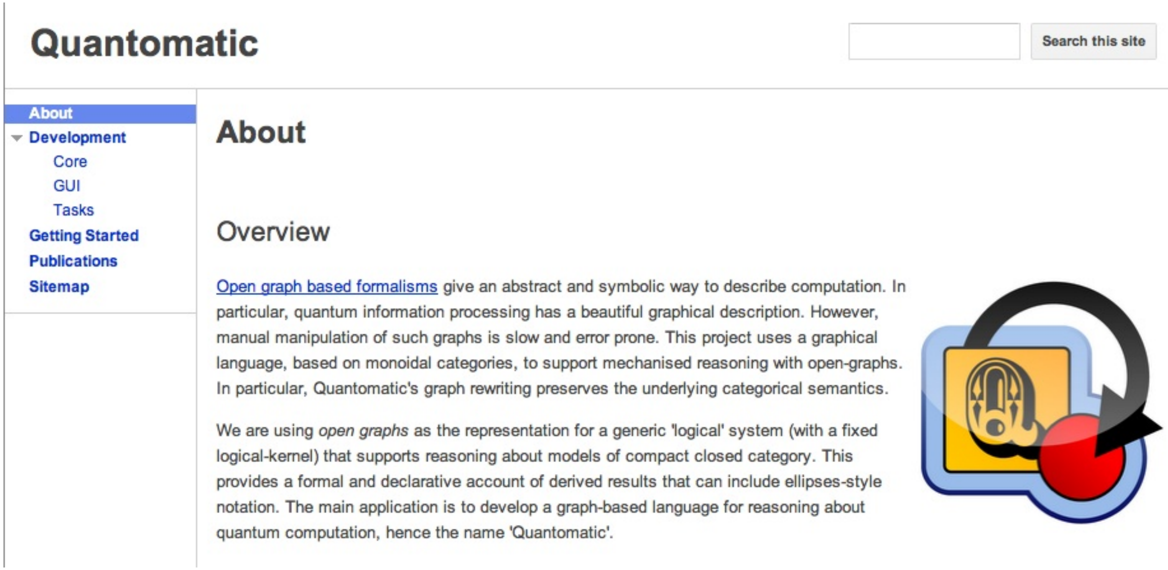
The children will win of course!

From: BC (2010) ***Quantum picturalism***. Contemporary Physics **51**, 59–83.

— *automation* —

— *automation* —

Exploiting discreteness and the 'logic of yanking':



Quantomatic

Search this site

- About
- Development
  - Core
  - GUI
  - Tasks
- Getting Started
- Publications
- Sitemap

About

Overview

Open graph based formalisms give an abstract and symbolic way to describe computation. In particular, quantum information processing has a beautiful graphical description. However, manual manipulation of such graphs is slow and error prone. This project uses a graphical language, based on monoidal categories, to support mechanised reasoning with open-graphs. In particular, Quantomatic's graph rewriting preserves the underlying categorical semantics.

We are using *open graphs* as the representation for a generic 'logical' system (with a fixed logical-kernel) that supports reasoning about models of compact closed category. This provides a formal and declarative account of derived results that can include ellipses-style notation. The main application is to develop a graph-based language for reasoning about quantum computation, hence the name 'Quantomatic'.

Dixon (Google), Duncan (Strathclyde), Soloviev (Cambridge), Kissinger, Merry, Quick, Zamdzhiev, BC (Oxford), – `sites.google.com/site/quantomatic/`

— *new physics* —

— *new physics* —

Well under way:

- **A new quantum formalism**

— *new physics* —

Well under way:

- **A new quantum formalism**
  - Many fragments/aspects of ours have been adopted by leading quantum foundations groups.

— *new physics* —

Well under way:

- **A new quantum formalism**
  - Many fragments/aspects of ours have been adopted by leading quantum foundations groups.

E.g. Lucien Hardy in arXiv:1005.5164:

"... we join the ***quantum picturalism*** revolution [1]"

[1] BC (2010) ***Quantum picturalism***. Contemporary Physics **51**, 59–83.

— *new physics* —

Well under way:

- **A new quantum formalism**
  – Many fragments/aspects of ours have been adopted by leading quantum foundations groups.

Promising:

- **Rigorous & convenient quantum field theory**
  – We captured common structure in QF and GR

— *new physics* —

Well under way:

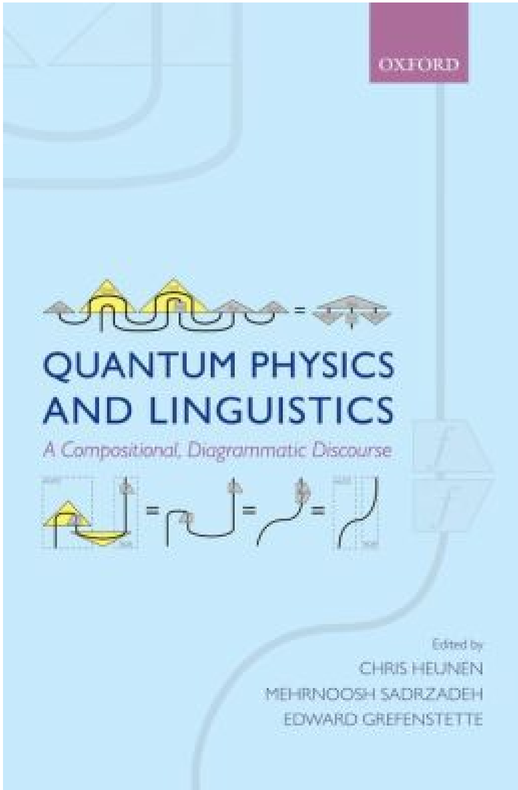
- **A new quantum formalism**
  - Many fragments/aspects of ours have been adopted by leading quantum foundations groups.

Promising:

- **Rigorous & convenient quantum field theory**
  - We captured common structure in QF and GR
- **Quantum gravity**
  - New programs based on our new Q-formalism have been launched by ourselves and others

and now for something completely different, ...

NATURAL LANGUAGE MEANING



BC, Mehrnoosh Sadrzadeh & Stephen Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. `arXiv:1003.4394`

FQXI ARTICLE September 29, 2013

Video Article: The Quantum Linguist

Bob Coecke has developed a new visual language that could be used to spell out a theory of quantum gravity—and help us understand human speech.

by Sophie Hebden

SCIENTIFIC AMERICAN™

Sign In / Register

Search ScientificAmerican.com

Quantum Mechanical Words and Mathematical Organisms

By Joselle Kehoe | May 16, 2013 | 10

QUANTUM LINGUISTICS Leap forward for artificial intelligence

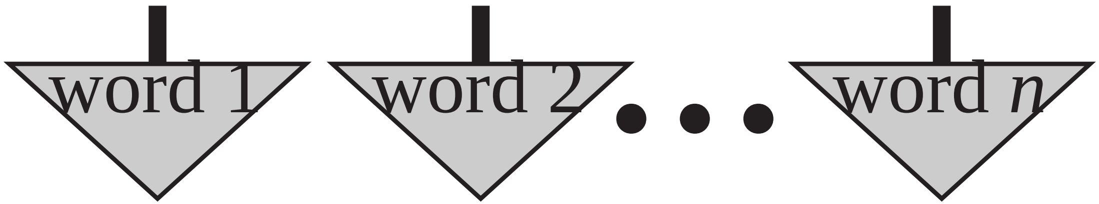
NewScientist

WEEKLY 11 December 2010

— *the from-words-to-a-sentence process* —

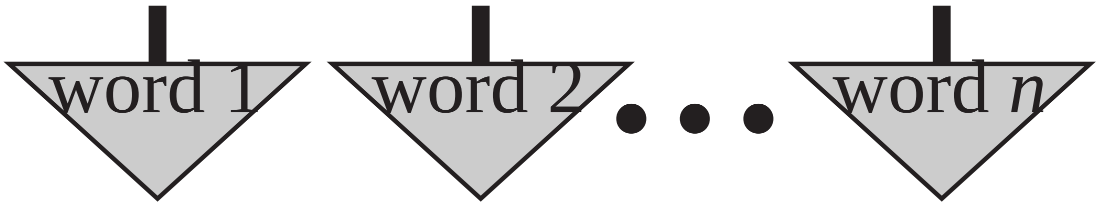
— *the from-words-to-a-sentence process* —

Consider meanings of **words**, e.g. as vectors (cf. Google):



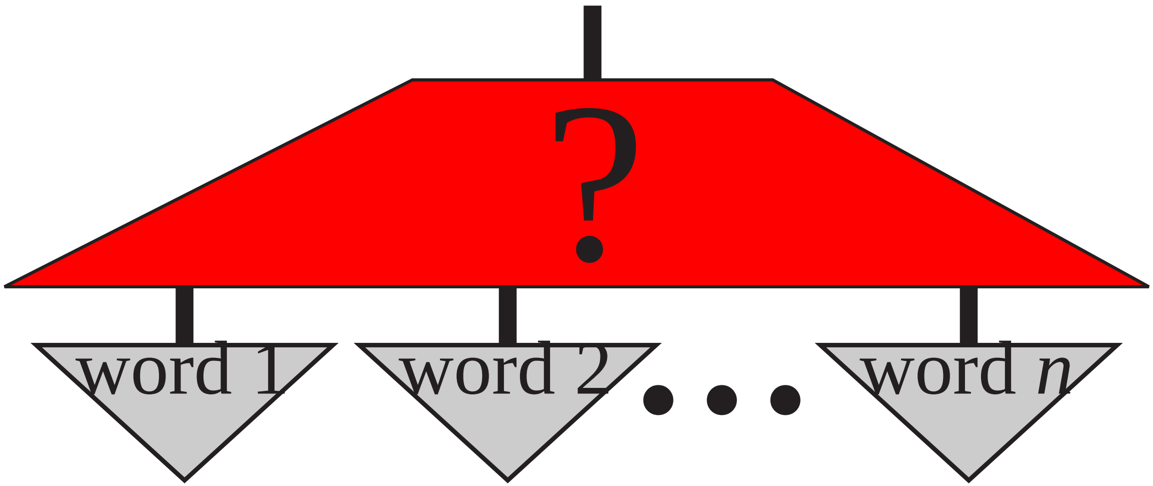
— the from-words-to-a-sentence process —

What is the meaning the **sentence** made up of these?



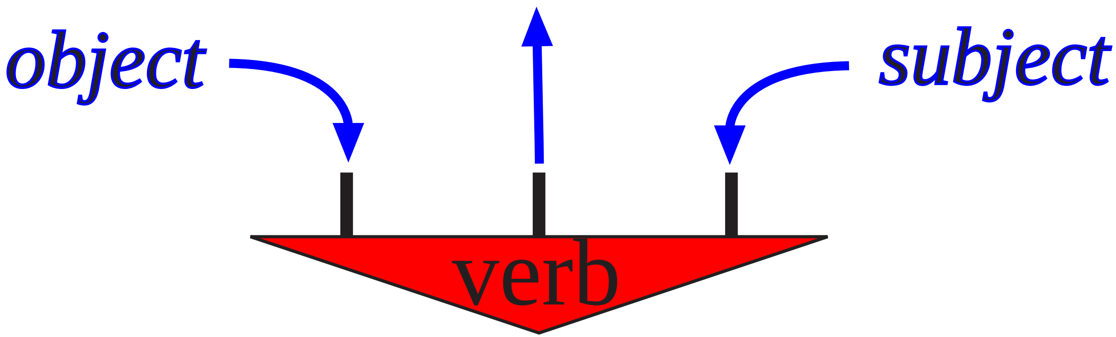
— the from-words-to-a-sentence process —

What is the meaning the **sentence** made up of these?



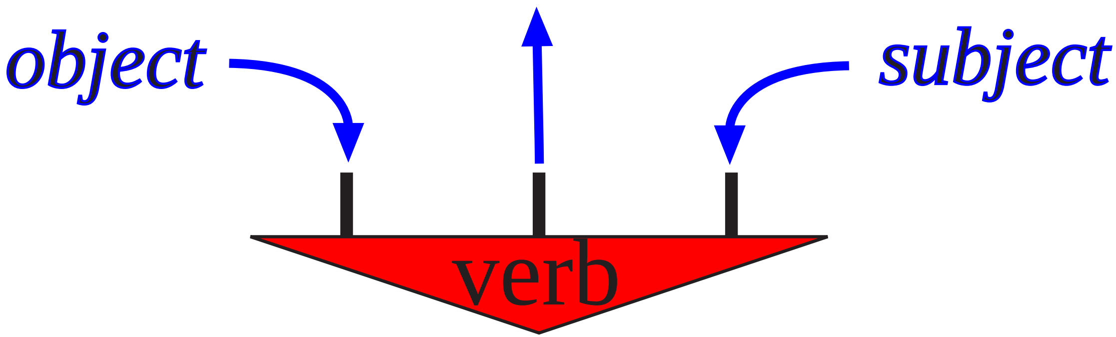
— ***the from-words-to-a-sentence process*** —

Information flow within a verb:

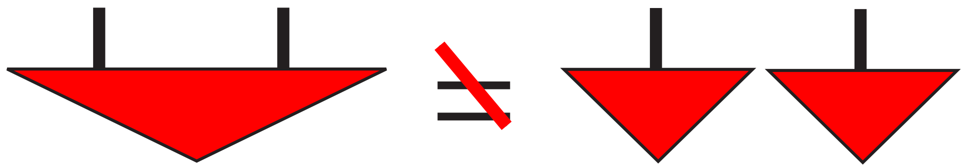


— the from-words-to-a-sentence process —

Information flow within a verb:



Again we have:



— *grammar* —

Lambek's Residuated monoids (1950's):

$$b \leq a \multimap c \Leftrightarrow a \cdot b \leq c \Leftrightarrow a \leq c \multimapinv b$$

or equivalently,

$$a \cdot (a \multimap c) \leq c \leq a \multimap (a \cdot c)$$

$$(c \multimapinv b) \cdot b \leq c \leq (c \cdot b) \multimapinv b$$

— *grammar* —

Lambek's Residuated monoids (1950's):

$$b \leq a \multimap c \Leftrightarrow a \cdot b \leq c \Leftrightarrow a \leq c \multimapinv b$$

or equivalently,

$$a \cdot (a \multimap c) \leq c \leq a \multimap (a \cdot c)$$

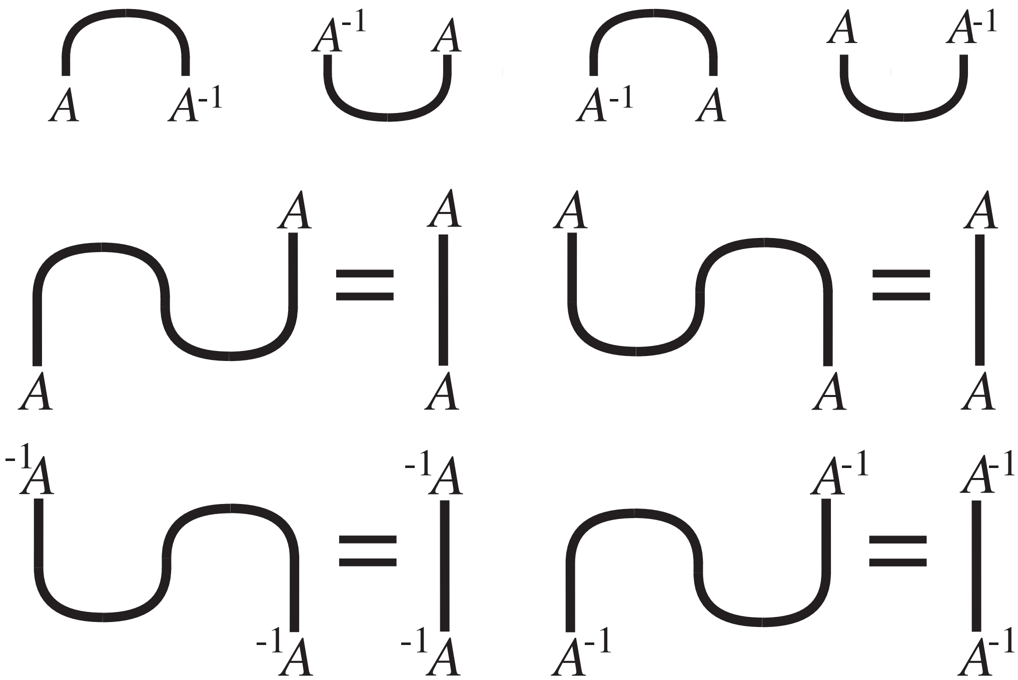
$$(c \multimapinv b) \cdot b \leq c \leq (c \cdot b) \multimapinv b$$

Lambek's Pregroups (2000's):

$$a \cdot {}^{-1}a \leq 1 \leq {}^{-1}a \cdot a$$

$$b^{-1} \cdot b \leq 1 \leq b \cdot b^{-1}$$

— *Lambek's pregoup grammar* —



— *Lambek's pregoup grammar* —

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

— *Lambek's pregoup grammar* —

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n$$

— *Lambek's pregoup grammar* —

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1$$

— *Lambek's pregoup grammar* —

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

— *Lambek's pregoup grammar* —

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

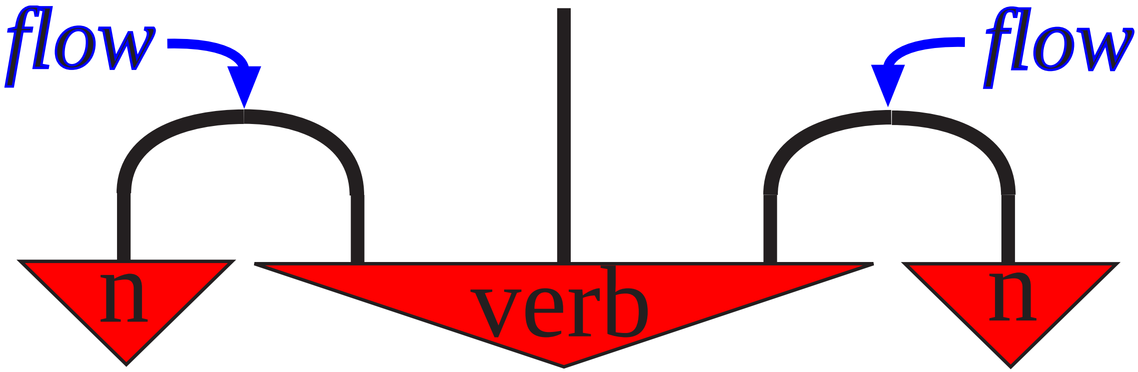
Diagrammatic type reduction:

— *Lambek's pregoup grammar* —

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

Diagrammatic meaning:



— *algorithm for meaning of sentences* —

— *algorithm for meaning of sentences* —

1. Perform type reduction:

$$(\textit{word type } 1)\ldots(\textit{word type } n) \quad \rightsquigarrow \quad \textit{sentence type}$$

— *algorithm for meaning of sentences* —

1. Perform type reduction:

$$(\textit{word type } 1)\ldots(\textit{word type } n) \quad \rightsquigarrow \quad \textit{sentence type}$$

2. Interpret diagrammatic type reduction as linear map:

$f ::$  $\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

— *algorithm for meaning of sentences* —

1. Perform type reduction:

$$(\textit{word type } 1)\ldots(\textit{word type } n) \rightsquigarrow \textit{sentence type}$$

2. Interpret diagrammatic type reduction as linear map:

$f ::$  $\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

3. Apply this map to tensor of word meaning vectors:

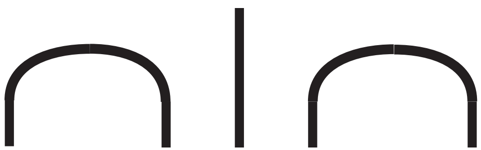
$$f\left(\overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n\right)$$

— *algorithm for meaning of sentences* —

1. Perform type reduction:

$$(\textit{word type } 1)\ldots(\textit{word type } n) \quad \rightsquigarrow \quad \textit{sentence type}$$

2. Interpret diagrammatic type reduction as linear map:

$f ::$  $\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

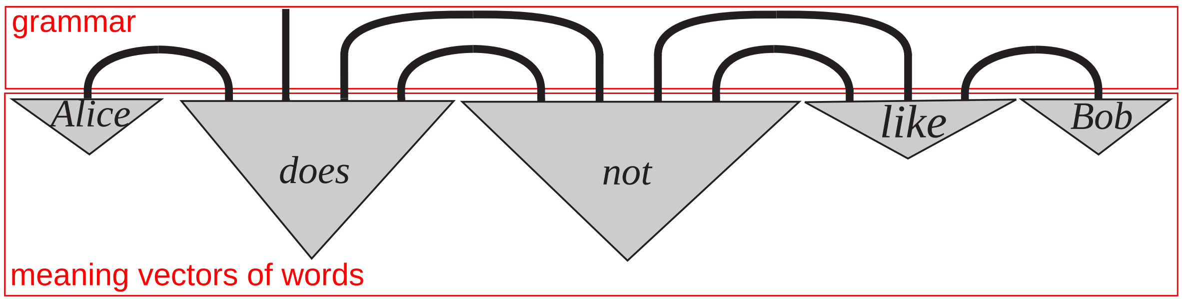
3. Apply this map to tensor of word meaning vectors:

$$f\left(\overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n\right)$$

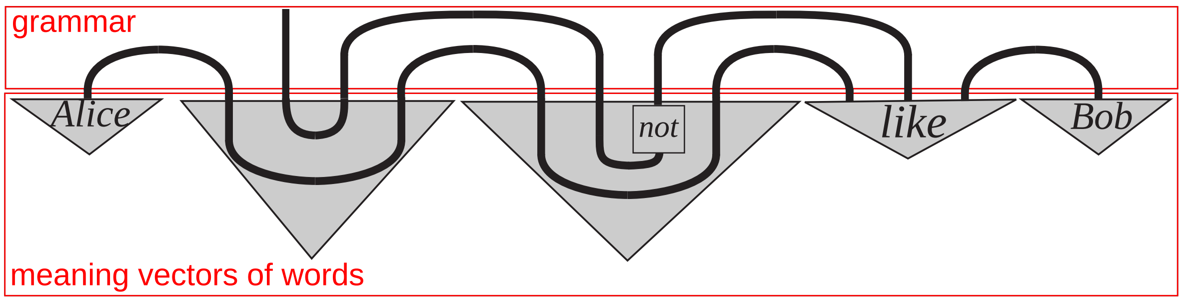
$\Longrightarrow$ **Outperforms all existing NLP methods both in speed as well as in scope for applicability**

— *grammer as meaning flow* —

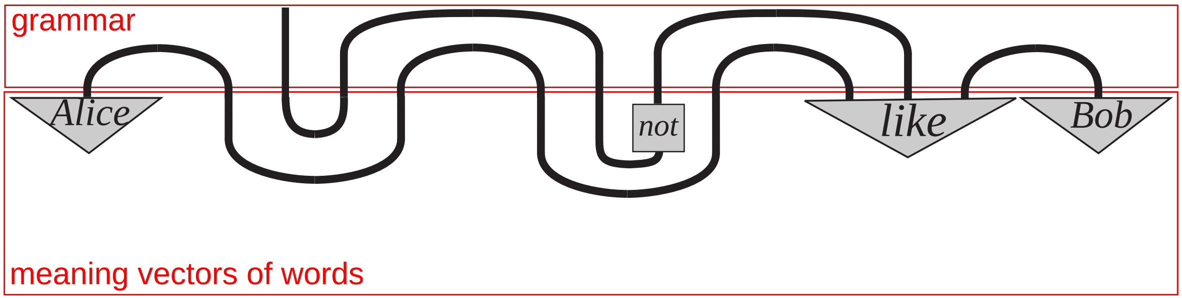
— *grammer as meaning flow* —



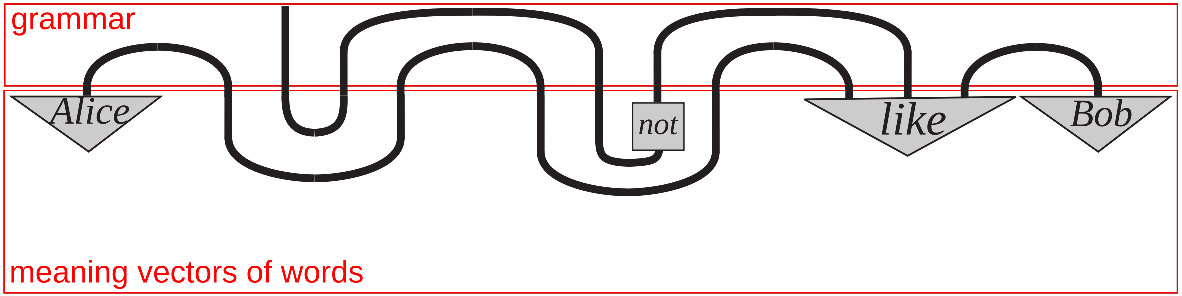
— *grammer as meaning flow* —

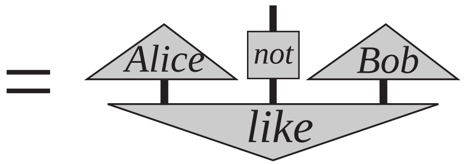


— *grammer as meaning flow* —

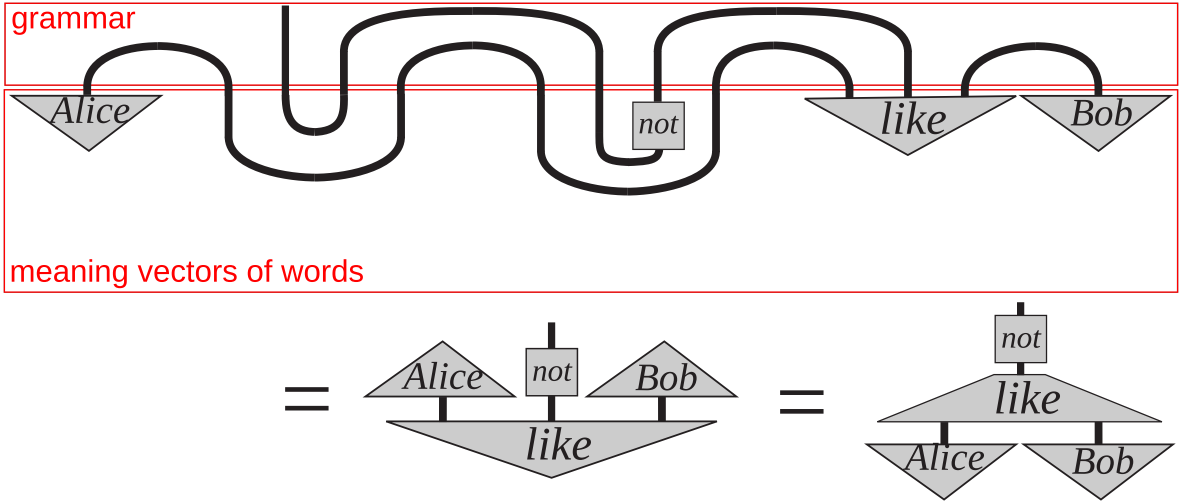


— *grammer as meaning flow* —





— *grammer as meaning flow* —



Using:

— *experiment: word disambiguation* —

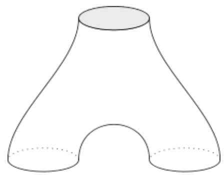
E.g. what is "saw"" in: "Alice saw Bob with a saw".

Model	High	Low	ρ
Baseline	0.47	0.44	0.16
Add	0.90	0.90	0.05
Multiply	0.67	0.59	0.17
Categorical (1)	**0.73**	**0.72**	**0.21**
Categorical (2)	**0.34**	**0.26**	**0.28**
UpperBound	4.80	2.49	0.62

Edward Grefenstette & Mehrnoosh Sadrzadeh (2011) *Experimental support for a categorical compositional distributional model of meaning*. Accepted for: Empirical Methods in Natural Language Processing (EMNLP'11).

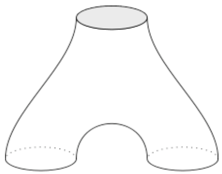
— QFT —

"Topological" QFT (Atiyah '88):

F ::  $\mapsto \quad f : V \otimes V \to V$

— QFT —

"Topological" QFT (Atiyah '88):

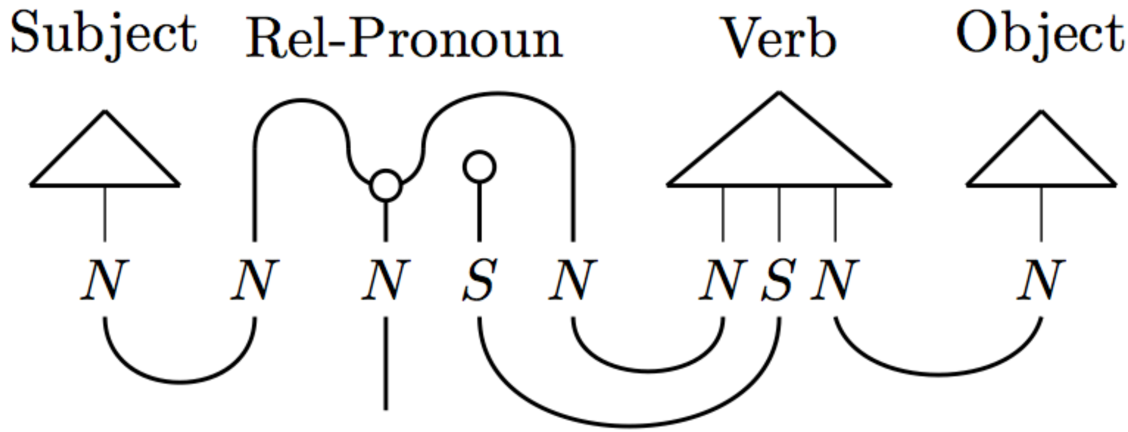
$F ::$  $\mapsto \quad f : V \otimes V \to V$

"Grammatical" QFT:

$F ::$  $\mapsto \quad \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

— *Frobenius algebras* —

Language-meaning:

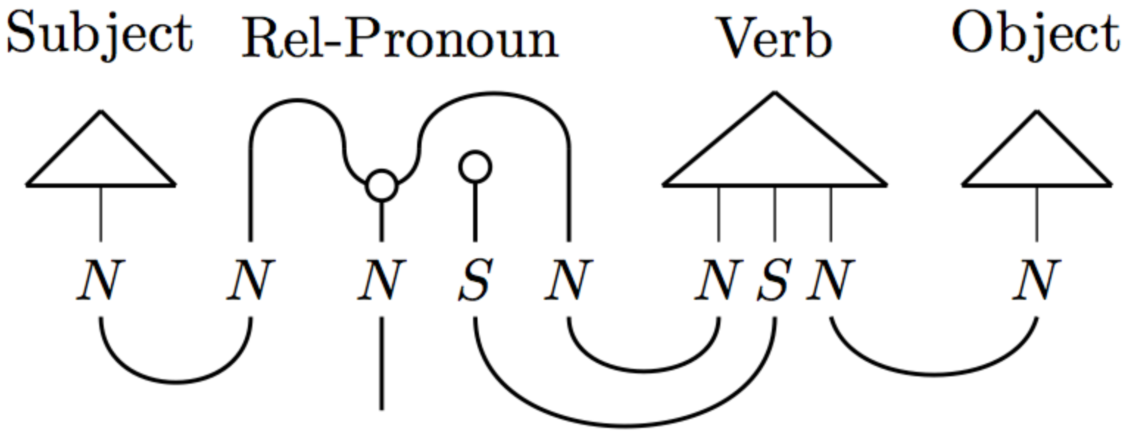


(the) woman who hates Bob

Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL '13.

— *Frobenius algebras* —

Language-meaning:

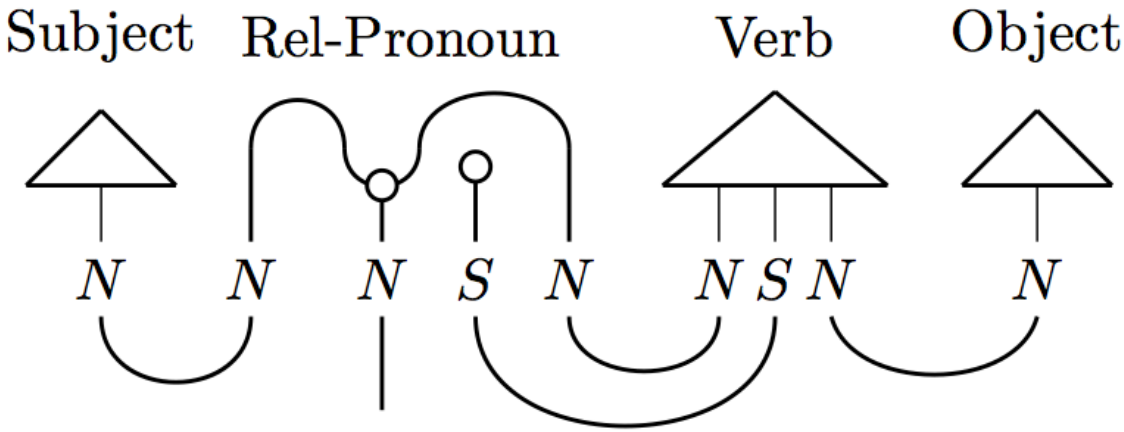


(the) woman who hates Bob =

Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL '13.

— *Frobenius algebras* —

Language-meaning:



(the) woman who hates Bob = Alice

Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL '13.