Reasoning with Probabilities

ESSLLI 2013 Course Outline

Joshua Sack*

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This document is a brief overview and outline of the course "Reasoning with Probabilities" held at the 2013 European Summer School in Logic, Language, and Information (ESSLLI 2013) in Dusseldorf, Germany, week 1 (August 5–9), slot 1. The course website is:

www.joshuasack.info/courses/2013/esslli-probability.html

Abstract

Both logic and probability provide a powerful tool for reasoning about uncertainty in diverse and dynamic environments. The goal of this course is to explore tools used by logicians, computer scientists, philosophers, and game theorists for modeling systems that employ logic and incorporate probability. Such tools will address logical frameworks of multi-agent uncertainty, clarify various conceptual issues (Aumann's agree to disagree result) and puzzles (such as Monty Hall puzzle). This course will focus on both important conceptual issues (e.g., Dutch book arguments, higher-order probabilities, and interactions between qualitative and quantitative uncertainty) and main technical results (e.g., completeness and decidability of probabilistic modal logics).

*Institute for Logic, Language, and Computation, University of Amsterdam. Website: http://www.joshuasack.info. Email: J.H.Sack@uva.nl

Motivation and description

Probability logic has been developed in philosophy, computer science, and game theory, often toward different goals, but using similar frameworks. This course aims to strengthen the understanding a student from one of these disciplines may have of reasoning about probabilities in general, while also gaining an appreciation for the utility of probabilistic reasoning in other disciplines. This course discusses a number of issues that are involved in philosophy, such as Dutch book arguments that motivate the general properties of probability, as well as a number of puzzles and conceptual issues, such as Aumann's agree to disagree result and the Monty Hall puzzle. Issues in game theory will be discussed such as higher-order probabilities, common p-belief, mixed strategies, and type spaces. Also stochastic interpretations of modal probability logics often used in computer science will be discussed, such as logics for deterministic and non-deterministic probabilistic transition systems. This course will also discuss different ways the interaction between qualitative and quantitative uncertainty are addressed; logics for non-deterministic probabilistic transition systems and probabilistic automata help us reason about non-determinism over probabilities while probabilistic epistemic logics help us reason about uncertainty of probabilities. This course will also cover dynamics by presenting logics for reasoning about how new information may result in changes to both subjective probabilities that individual agents in a group have about the state of the world and qualitative uncertainties the agents have. Variations of this dynamic probabilistic epistemic logic will be discussed as well as some technical goals, such as completeness and decidability theorems.

Tentative Rough Outline

Day 1: Background and basic concepts

Day 2: probabilistic modal logic

Day 3: Dynamic Probabilistic Epistemic Logic

Day 4: Higher-order probability and puzzles

Day 5: Summary and concluding remarks

Prerequisites

Although this course has an interdisciplinary goal aiming to introduce students from one discipline to methods in another, and although the course is self-contained with many basic concepts of probability theory (e.g., measure spaces) being explained, it is considered an advanced course because it focuses and builds on underlying logical structures and tools for reasoning about probabilities that transcend individual disciplines. It will be expected that students have had previous exposure to probabilistic models of uncertainty as well as epistemic logic. Exposure to dynamic extensions of epistemic logic (such as public announcement logic) would be also be helpful.

Extended (tentative) outline

Day 1: Background and general concepts.

This lecture will open with an example of a probability result called "Agreeing to Disagree" (see [Aumann, 1976]), and then will review some basic epistemic logic, probability and measure theory, and Dutch book motivation for probability axioms and updates. For a more extensive discussion, you may want to look at [Halpern, 2003, van der Hoek et al., 2007] for *epistemic logic* and [Billingsley, 1995] for *probability theory*. The Dutch book literature involves synchronous Dutch book that relate the laws of probability to betting games, and diachronic Dutch book that relate updating of probabilities to betting games. A good overview of *synchronic* Dutch book arguments is [Hajek, 2008], and of *diachronic* Dutch book arguments is [Skyrms, 1987]. For basic concepts in game theory, you might want to look at [Osborne and Rubenstein, 1994] if needed. For a general overview of logic and probability, see the Stanford Encyclopedia of Philosophy article [Demey et al. 2013].

Day 2: Probabilistic (modal) logic

This lecture will cover probabilistic propositional logic, probabilistic modal logic (which makes use of higher-order probabilities), and logics that combine qualitative and quantitative uncertainty. For probabilistic propositional logic, see [Fagin et al., 1990]. For probabilistic modal logic, we may investigate the work of [Larsen and Skou, 1991], which was motivated by computation, and [Heifetz and Mongin, 2001], which was motivated the involvement

of Harsanyi types in game theory. For the interaction between quantitative and qualitative uncertainty see [Sack and Zhang, 2012] in the setting of computation, and [Fagin and Halpern, 1994] for probabilistic epistemic logic. The plan in to cover language, semantics, and for some of these languages, proof system, weak completeness theorems, and computational complexity results of some of these logics. The completeness and complexity results may make use of some techniques from epistemic logic. See [Halpern and Moses, 1992] for a survey.

Day 3: Dynamic probabilistic epistemic logic

This lecture will cover logics that mix probability, epistemics, and time. Recommended reading is probabilistic dynamic epistemic logic: [van Benthem et al., 2009], where three different types of probabilities are involved an updating process. A setting involving potentially continuous models is given in [Aceto et al. 2011].

Day 4: Higher-order probabilities and Puzzles.

This lecture will discuss the utility of higher-order probabilities (probabilities of probabilities), type spaces, as well as common p-belief, logic for mixed strategies, as well as some popular puzzles on logic. Suggested reading is [Harsanyi, 1967] for type spaces, [Kajii and Morris, 1995, Monderer and Samet, 1989] for common p-belief, and [Sack and van der Hoek, 2013] for mixed strategies. This lecture will describe some popular puzzles on probability. Puzzles will likely include Monty-Hall, Two-Envelope, Sleeping Beauty, Absent-minded driver, Cable Guy.

Day 5: General remarks

Time permitting, we may discuss probabilistic belief dynamics [Baltag and Smets, 2008].

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